CHAPTER 2

Foundations of Conservation Principles

2.1 Instructional Objectives

After completing this chapter, you should be able to do the following:

- Identify an extensive property that can be counted in a system of interest.
- Appropriately define a system, system boundary, and surroundings for a system of interest.
- Identify whether a given system is described by a continuous time period or a discrete (finite) period, and if the latter, specify a time period of interest.
- Know the theory and scope of the conservation laws.
- Explain the differences between an accounting equation and a conservation equation.
- Explain the differences among algebraic, differential, and integral accounting and conservation equations, and select an appropriate equation for a system.
- Describe the differences among open, closed, and isolated systems; reacting and nonreacting systems; systems with and without energy interconversions; and steady-state and dynamic systems. Correctly describe systems using these definitions.

2.2 Introduction to the Conservation Laws

Conservation—the preservation of a physical quantity during movement, transformation, or reaction—is a fundamental concept in engineering and science. Along with the second law of thermodynamics (the disorder in the universe spontaneously increases), a handful of conservation principles provide the governing laws for virtually all physical behavior. Engineers mathematically describe these laws in conjunction with initial or boundary conditions. The foundations of many fields of engineering, including bioengineering, are based on understanding and applying conservation laws.

For centuries, scientists and engineers have recognized that certain physical quantities could be described differently than other physical quantities. For example, Sir Isaac Newton's second law of motion, developed in the late 1600s to relate the net force on an object to its mass and acceleration, is a special case of the law of conservation of linear momentum. Formulated in 1845, Kirchhoff's current law, which

states that the total charge flowing into a node must equal the total charge flowing out of the node, is an equation developed on the concept of conservation of charge. Although these specific applications of the conservation laws were developed centuries ago, scientists and engineers more recently have generalized the conservation of multiple extensive properties into a few governing conservation laws. These laws can be applied to total mass, mass and moles of elements, linear and angular momentum, net electrical charge, and total energy. These laws have become axioms that serve as the fundamental basis for problem solving across engineering disciplines.

The conservation laws can be mathematically described by and formulated into conservation equations, which are described in Section 2.6. Conservation laws have been used in a wide variety of applications across all fields of engineering:

- Refining crude oil into gasoline
- Finding bending moments and loads in building structures
- Designing and constructing circuits and computers
- Estimating ground-water contamination
- Designing and producing microchips
- Modeling the carbon cycle in the environment
- Designing and building aircraft
- Developing life support systems

In this text, we explore the application of the conservation laws to many systems, such as the following:

- Human kidney
- Blood circulation
- Cell metabolism
- Cell membrane ion pumps
- Human exertion
- Air flow in lungs
- Platelet adhesion
- Hypothermia
- Hemodialysis
- Batteries
- Basal metabolic rate
- Stenotic blood vessels
- Electrical circuits
- Acid/base buffering
- Pumping heart
- Membrane potentials
- Biomaterials
- Tissue engineering
- Medical device design

Not all extensive physical properties are conserved. In other words, some extensive properties are created or destroyed during changes in or to the system. Those that are not conserved must be described using an accounting equation, a more generalized version of the conservation equation. This textbook is structured to cover

how each governing conservation law is applied and provides numerous examples of the application of accounting and conservation equations to the diverse fields in bioengineering.

2.3 Counting Extensive Properties in a System

Counting objects or quantities of extensive properties is the basis of **accounting**. In engineering systems, the counting of objects can be reduced to a few **accounting statements**. Specifically, many extensive properties—but no intensive properties—can be counted using accounting statements. Recall that extensive properties depend on the size of the system (see Section 1.5.1). Below is a list of extensive properties that can be counted:

- Total mass
- Mass of individual species
- Mass of individual element
- Total moles
- Moles of individual species
- Moles of individual element
- Total energy
- Thermal energy
- Mechanical energy
- Electrical energy
- Net electrical charge
- Positive electrical charge
- Negative electrical charge
- Linear momentum
- Angular momentum

All the extensive properties listed above can be counted in accounting equations, but only a subset of these extensive properties is always conserved. Below is a complete list of extensive properties that are conserved in all situations (except nuclear reactions):

- Total mass
- Mass of individual element
- Moles of individual element
- Total energy
- Net charge
- Linear momentum
- Angular momentum

To set up an accounting or conservation statement, three initial steps must be completed:

1. The *extensive property* to be counted must first be specified.
2. The *system* and its *surroundings* must be defined by specifying a *boundary*.
3. The *time period* must be specified.

In other words, you as the problem-solver must select and state the property of interest, system, and time period for the problem at hand.

2.3.1 Specifying the Property

The first step in setting up an accounting or conservation statement is to specify the extensive property. Typical properties include a type of mass, energy, charge, or momentum.

You are probably familiar with the concept and application of using an organized accounting scheme to keep track of items or objects. For example, the manager at a university bookstore needs to count all the books, school supplies, and clothing items that enter and leave the bookstore. The manager wants to keep track of each type of item and the type of client that is purchasing the various items, as shown in Table 2.1. Developing a simple spreadsheet like the one depicted in Table 2.1 allows the manager to keep track of the inventory items that can be counted. Mathematical equations that capture the same process used by the manager to track and count items are called accounting statements.

When we consider this example, some features of accounting equations become apparent. First, **the same property must be counted in all terms in a single equation.** If you were writing an accounting equation for the total number of books in the university bookstore, school supplies such as rulers and notebooks would not be included. However, if you were interested in developing a "total clothing" accounting equation, you would include both the number of T-shirts and the number of sweatshirts in your count. When accounting a particular property, the units of all the items must be the same. For example, when accounting mass, all quantities must be a dimension of mass; when accounting energy, all quantities must be a dimension of energy; when accounting a particular chemical species, all quantities must be that specific chemical species; and so on for other extensive properties.

The extensive property to be counted must be present in the system of interest. In other words, there must be some extensive property contained in the system at some time. It makes no sense to consider a property that is not contained in the system during the time period. As is discussed later, extensive properties may also enter or leave the system, or be generated or consumed by the system.

2.3.2 Specifying the System

A **system** consists of matter identified for investigation. The system is set apart from the **surroundings**, defined as the remainder of the universe (Figure 2.1). The **system boundary** separates the system from the surroundings.

TABLE 2.1

Inventory at University Bookstore

Date	Item	Inventory at bookstore	Delivered to bookstore	Sold to students	Sold to faculty and staff	Sold to nonuniversity individuals
8/20	Books	13,000	800	4,900	100	0
8/20	Supplies	1,000	150	300	25	10
8/20	T-shirts	400	0	15	25	100
8/20	Sweatshirts	400	0	15	25	100
8/21	Books	8,800	200	4,000	100	5
8/21	Supplies	815	0	300	0	0

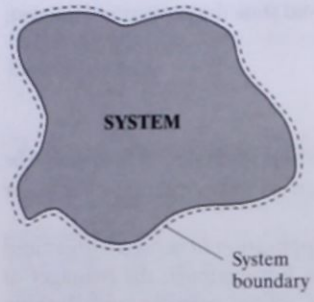


Figure 2.1
System and surroundings.

Defining the system of interest before you begin to solve a problem is important, since it may change your conditions and assumptions. **Systems are defined by the problem solver based on the needs of the problem.** The system can be quite large or quite small. For example, when investigating a biochemical reaction in the human body, the system may be defined as the entire body, a specific organ in the body, one cell in that organ, an organelle within the cell (e.g., nucleus), or in a number of other ways (e.g., an arbitrary volume of cytoplasm in the cell) (Figure 2.2).

The system could be already encapsulated as a unit, such as a bone, battery, or bioreactor. The system could also be dispersed or poorly defined in a physical form. Examples of this include the vascular system, alveoli in the lungs, or rivers in a country.

The system is determined and defined by the system boundary. Often, the system boundary is drawn in a way to "cut across" the transfer of an extensive property into or out of the system. Sometimes, the system boundary is selected to capture a change in the system. The system boundary should establish or isolate the system for investigation. In this text, the system boundary is indicated by dotted lines.

System boundaries are of two types. The first type of boundary is real and tangible, meaning the boundary naturally exists and has a definitive border. Such boundaries may enclose the entire object of interest. Some examples are the walls of a glass beaker, where the system is the liquid contained in the beaker; the casing of a total artificial heart, where the artificial heart is the system; or the plasma membrane of a cell, where the cell is the system. The second type of boundary is arbitrarily defined by the problem solver. It can be a sectional area of an object that accurately models the larger object or one that captures all relevant elements including the movement of the property of interest across the boundary. An example of an arbitrary boundary is a 1 cm $\times$ 1 cm patch of the villi wall; modeling the villi wall as such a patch may give insight into how all the villi in the small intestine act. A second example is isolating two malfunctioning units of a seven-unit bioreactor process. Other examples are a hypothetical line in space or a plane in a vessel. For the list of systems in Figure 2.2, the system boundaries are defined by the skin, the organ wall, the cell membrane, the nuclear membrane, or some hypothetical boundary line in the cytoplasm.

Drawing a picture and labeling the system and its surroundings is critical in this process. While it can make sense to draw a picture that is representational, "black box" models may also work. Representational diagrams are important when the geometry or physical layout of the system is important. Block or box diagrams can be used to capture simple or complex systems when geometric concerns are not important. For most problems in this text, box diagrams suffice and are preferred.

In the university bookstore example, the problem can be set up with the bookstore as the system of interest (Figure 2.3). The surroundings include the rest of the universe, except the bookstore. A dotted line is the system boundary and separates

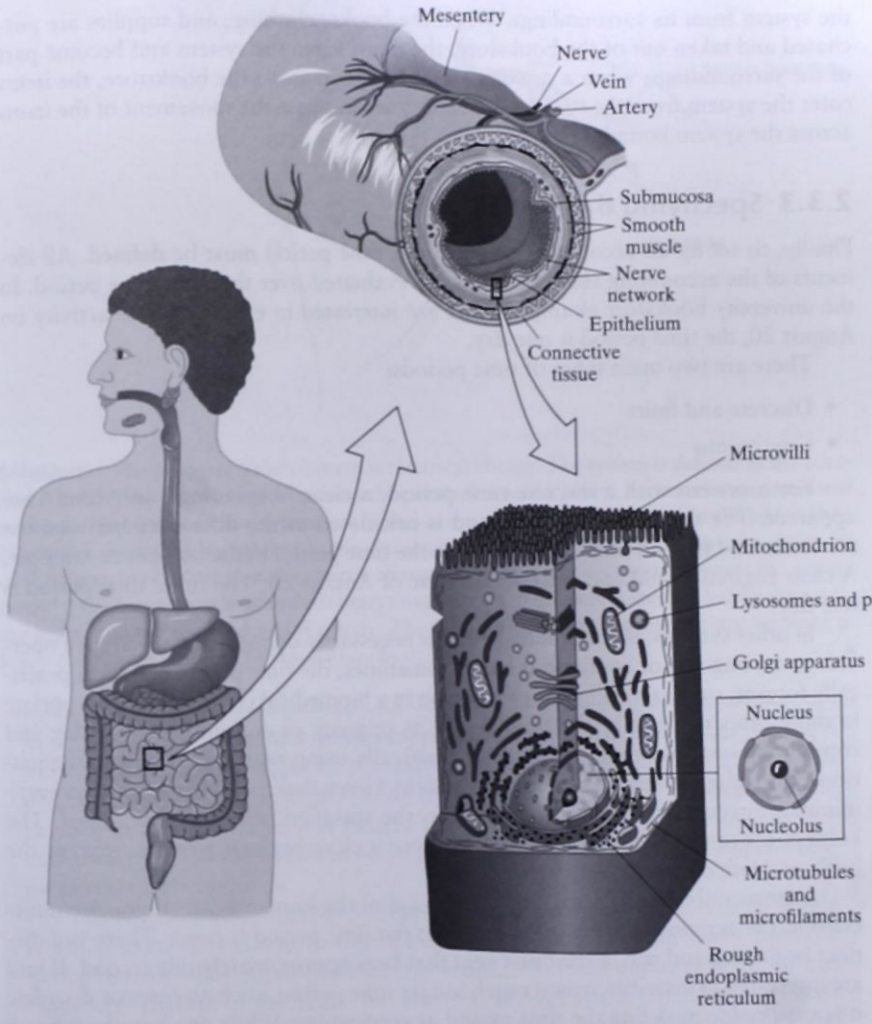


Figure 2.2
Systems of various scales. (*Source:* Silverthorn DU, *Human Physiology*, 2d ed. Upper Saddle River, NJ: Prentice Hall, 2001.)

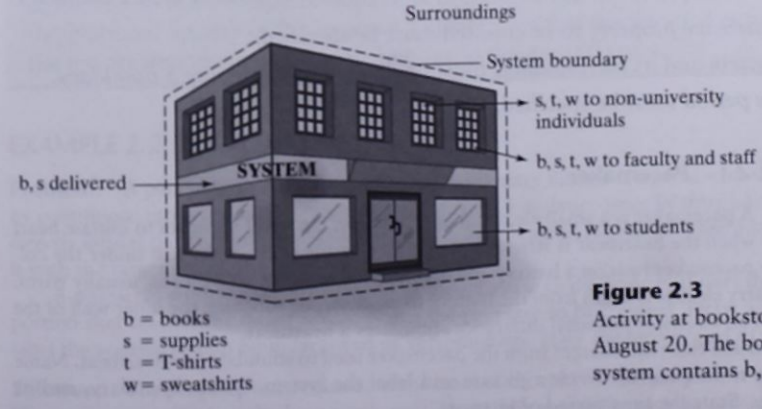


Figure 2.3
Activity at bookstore on August 20. The bookstore system contains b, s, t, and w.

the system from its surroundings. When the books, clothing, and supplies are pur chased and taken out of the bookstore, the items leave the system and become par of the surroundings; when a delivery truck brings items to the bookstore, the item enter the system from the surroundings. Arrows indicate the movement of the item across the system boundary.

2.3.3 Specifying the Time Period

Finally, to set up an accounting statement, **a time period must be defined.** All ele ments of the accounting statement must be evaluated over the same time period. I the university bookstore example, if we are interested in evaluating the activity o August 20, the time period is one day.

There are two main types of time periods:

- Discrete and finite
- Continuous

For a system with a discrete time period, a clear "beginning" and "end" ar apparent. The time period is finite and is calculated as the difference between th start time and the end time. An example is the time period in the bookstore example A clear beginning and end exist on the date of August 20. The finite time period i one day.

In other systems, no beginning or end is necessarily defined, and the system oper ates on an ongoing or continuous basis. Sometimes, the time period can seem practi cally forever, such as the lifetime of a person in a biomedical context. It is appropriat in these cases to describe the time period as ongoing or continuous. Ongoing and continuous systems are described mathematically using rates and differential equa tions. Consider the human kidneys as a system. Given that the kidneys function every minute of every day for an entire lifetime, the specified period is continuous. The kidneys operate on an ongoing basis, without a clear beginning or end (except the birth or death of a person).

A more ambiguous case is the time period of the human heart. If you are inter ested in the activities during one beat, then the time period is finite. There is a dis tinct beginning and end in the heart beat that lasts approximately one second. If you are interested in activities over a much longer time period, such as years or decades, it is reasonable to define the time period as continuous. While one heartbeat has a distinct beginning and end, the heart continues beating over an indeterminate span of time.

In review, to be able to write an accounting statement, three items are necessary:

- The *extensive property* to be counted must be specified.
- The *system* and its *surroundings* must be defined by specifying a *boundary*.
- A *time period* must be specified.

EXAMPLE 2.1 Pacemaker

Problem: A pacemaker is a small electrical unit that uses electrical impulses to initiate heart contraction when the heartbeat is irregular. Implanted in the chest cavity just under the col larbone, the pacemaker runs on a battery and is connected to the heart by leads, usually wires. The wires carry electric charges from the battery to an electrode touching the inner wall of the heart, where an electrical potential difference stimulates a heartbeat.

Consider the electrical charges from the pacemaker used to stimulate one heartbeat. Name the property to be counted. Draw a picture and label the system, system boundary, and its surroundings. State the time period of interest.

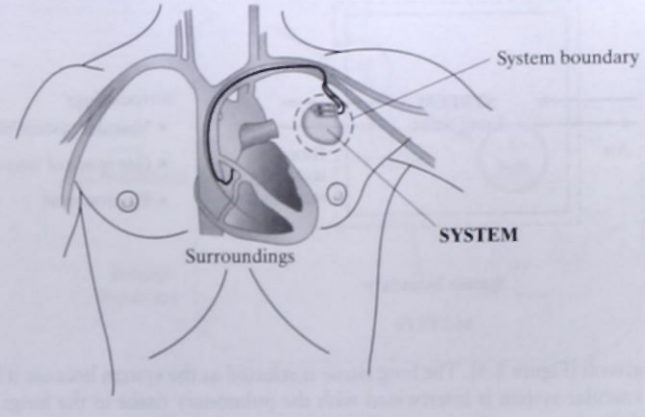


Figure 2.4
Pacemaker connected to heart.

Solution: The property to be counted is electrical charge. The system is defined as the pacemaker (excluding the wire running from the pacemaker to the heart), because we are interested in the charge contained in the pacemaker and in the charge that moves out of the pacemaker. Figure 2.4 shows the pacemaker as the system, and the boundary is defined by the perimeter of the pacemaker. Note that the electric charge that leaves the pacemaker crosses the system boundary. The surroundings include everything outside of the pacemaker, including the wires, the heart, and the remainder of the body. The time period is one heartbeat, since we want to know the charge required to initiate one beat. ■

In 1950, electrical engineer John Hopps accidentally discovered that a cooled heart could be started again by stimulating it with electrical impulses. With this discovery, Hopps inadvertently made the world's first cardiac pacemaker. The device's large size and limited battery life were impractical for use in patients, and not until 1960 did a team of surgeons successfully insert the first implantable pacemaker into a human.

The engineer who improved Hopps' design was Wilson Greatbatch. While working to create a circuit to record fast heart sounds, he accidentally used the wrong resistor—but discovered that it generated electrical impulses in the unique "lub-dub" rhythm of the heart. For the next several years, Greatbatch worked to develop a long-lasting, corrosion-free lithium-ion battery and to reduce the device to the size of a matchbook. Greatbatch's innovation has helped millions of patients maintain a regular heartbeat, allowing them to regain a normal quality of life and to live a lifespan comparable to those of healthy individuals. In 1985, the National Society of Professional Engineers named the pacemaker as one of the ten greatest engineering contributions in the past 50 years.

EXAMPLE 2.2 Drug Delivery

Problem: A patient with chronic obstructive pulmonary disease is treated with theophylline by continuous intravenous (IV) infusion at a rate of 0.5 mg/min. Since IV drug administration directly infuses the drug into the bloodstream, the blood carries the theophylline and exchanges it with the target organ, the lungs.

A physician studying the distribution of theophylline in the body evaluates the dose proportion that reaches the target organ. Name the property to be counted. Draw a picture and label the system, system boundary, and its surroundings. State the time period of interest.

Solution: The property to be counted is the mass of theophylline in the patient's lung tissue. The system is defined as the lung tissue outside the blood vessels, such as cells that line the

can be counted. In this text, we discuss how accounting equations are used to track the extensive properties of:

- Total mass
- Mass of individual species
- Mass of individual element
- Total moles
- Moles of individual species
- Moles of individual element
- Total energy
- Thermal energy
- Mechanical energy
- Electrical energy
- Net electrical charge
- Positive electrical charge
- Negative electrical charge
- Linear momentum
- Angular momentum

Note that volume is not on this list. Compressible fluids (especially gases) may invalidate volume-based accounting equations. If volumes are given, use density to convert to mass values and then use a mass accounting equation.

A specialized version of the accounting equation tracks only extensive properties and the rates of extensive properties that are conserved. By definition, a **conserved property** is one that is neither created nor destroyed. The **conservation law** states: **When an extensive property is conserved, that property is neither created nor destroyed despite changes in or to the system or the surroundings.** Not all extensive properties are conserved, but several key ones are as follows:

- Total mass
- Mass of individual element
- Moles of individual element
- Total energy
- Net charge
- Linear momentum
- Angular momentum

The **conservation equation** is a mathematical description of the movement and accumulation of an extensive property in a system of interest, eliminating any terms that account for creation or destruction of an extensive property. When an extensive property is conserved, the amount or rate of that extensive property is unchanged in the universe at all times (except for nuclear reactions), where the **universe** is defined as the system and its surroundings. Thus, there is no net change of the amount of that conserved extensive property in the universe. In other words, the total amount of a conserved extensive property in the universe is a constant.

This is not the same as the common physics definition of conservation, in which the quantity of the extensive property would be "conserved" and therefore constant in the system. Instead, conserved in this context means that the sum total of the extensive property remains the same in the universe but may be transferred between the system and its surroundings.

While it cannot be created or destroyed in a system, a conserved extensive property can be exchanged with the system's surroundings. Changes in the amount of a conserved property in a system can happen only by a one-for-one exchange with its surroundings. A conserved extensive property can also accumulate in or be depleted from a system.

Consider the conservation of total mass: The amount of total mass added to a system is equivalent to the amount of total mass that the system's surroundings gave up. In this case, the net amount of total mass in the system increased, the net amount of total mass in the surroundings decreased, and the amount of total mass in the universe is unchanged. As an example, consider a system defined to be a person. If she or he eats a candy bar, the mass of the system increases by the mass of the candy bar. However, the mass of the surroundings decreases by the same amount, since the candy bar has crossed the system boundary and is no longer included in the surroundings. The total mass of the universe (i.e., person, candy bar, and everything else) is unaffected by the transfer of the candy bar into the system.

In contrast to total mass, species mass is not conserved and should be described using an accounting equation. When a reaction occurs in a system to produce a particular chemical species, the net amount of that species mass increases—both in the system and in the universe. Since there is a net change in species mass in the universe, the species mass is not conserved. As an example, consider the production of acetaminophen (Tylenol) from other chemicals in a reactor in a pharmaceutical plant. Once a new batch of acetaminophen is made through chemical reactions, there is more acetaminophen in the system and the universe.

It is important to understand the criteria that separate conserved and nonconserved properties. To reiterate, all extensive properties that can be counted can be described with an accounting equation. Only those extensive properties that are neither created nor destroyed in the universe may be described using a conservation equation.

Mathematically, we illustrate both these concepts—accounting and conservation—in governing equations:

Accounting Equation

$$\text{Input} - \text{Output} + \text{Generation} - \text{Consumption} = \text{Accumulation} \quad [2.4\text{-}1]$$

Conservation Law Equation

$$\text{Input} - \text{Output} = \text{Accumulation} \quad [2.4\text{-}2]$$

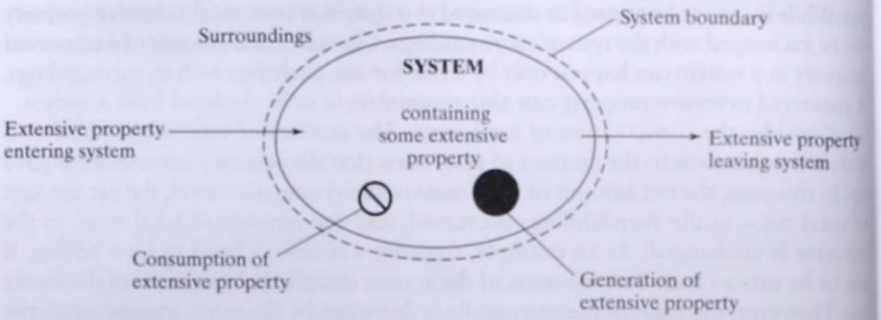
The difference between the final and initial amounts in the system mathematically describes the Accumulation term in both the accounting and conservation equations:

$$\text{Accumulation} = \text{Final Condition} - \text{Initial Condition} \quad [2.4\text{-}3]$$

Figure 2.9 schematically represents the generic accounting equation situation for any extensive property. For a system in which the extensive property is conserved, no generation or consumption occurs.

2.4.1 Input and Output Terms Describe Exchange of Extensive Property

The **Input** and **Output** terms capture the exchange or transfer of an extensive property into and out of a system. The Input and Output terms describe all exchanges in which the amount of extensive property transferred to or from the system is equal to that lost or gained by the surroundings, respectively. The terms can also describe situations in which the quantity of extensive property exchanged is balanced across



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the system boundary. Input and Output terms are usually represented by arrows in system diagrams. Exchange or transfer of extensive property can occur by several different modes.

- An extensive property can be transported by *bulk movement*. In this case, an extensive property, often as material, is physically transferred or moved across the system boundary. The extensive property can be moved into a system, out of a system, or both as long as the material moves across the system boundary. Mass, energy, charge, and momentum can be exchanged by bulk movement. Bulk movement can be of an extensive property or of the rate of an extensive property. For example, if we define a system as a baseball catcher's mitt and the air in proximity surrounding the glove, moving a baseball across the boundary into the mitt system adds mass, energy, and momentum to the system through the bulk movement of the baseball.
- An extensive property can be transferred by *direct contact*. In this case, an extensive property is transferred to or from an object that physically touches the system without the object or any material crossing the system boundary. Energy and momentum can be exchanged through direct contact. An example is the transfer of heat by a heating jacket touching the system boundary around a bioreactor. The heat jacket does not transfer energy through movement of a hot material, such as a molten liquid; instead, the heat is energy moving down a temperature gradient. In this case, the energy gained by the bioreactor through heat is equal to the energy lost by the surroundings (the heating jacket).
- An extensive property can be transferred through *nondirect contacts*. In this case, the system is acted on across a distance. The most frequently applied type of this transfer is a potential field. Energy and momentum can be transferred through nondirect contacts. For example, when considering the momentum conservation law, gravitational forces are included in the Input and Output terms. The gravitational forces that act on a system are balanced by other forces in the surroundings.

The last two types of exchanges (direct and indirect contacts) can be a bit more abstract and difficult to visualize than bulk transfer. In summary, the Input includes all extensive property that is added to the system across the system boundary. Likewise, the Output term describes the amount of extensive property that is lost by the system across the system boundary.

EXAMPLE 2.5 Currency Circulation at a Bank

Problem: During a normal workday, a local bank circulates physical currency as it handles customer deposits and withdrawals. Create a model that describes how much money passes through the bank from 8 AM to 5 PM assuming that the total money in the bank is unchanged

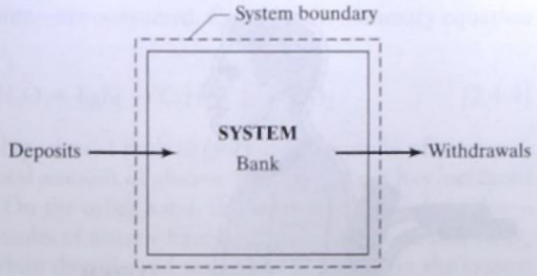


Figure 2.10
Currency transfer in a bank.

at the end of the stated business hours. Describe what Input and Output terms may be present in the accounting equation.

Solution: For this model, the extensive property is dollars. The system is the bank because it is important to model the currency that moves into and out of the bank (Figure 2.10). The system boundary is the physical exterior of the bank.

The Input term is all currency deposits that customers make between 8 AM and 5 PM, while the Output term is all currency withdrawals made in the same period. The transfer of money across the system boundary occurs through physical withdrawals and deposits of dollar bills (and cents), which can be considered *bulk* transfer. ■

EXAMPLE 2.6 Ice Pack Treatment for Sprained Ankle

Problem: A track athlete uses an ice pack to relieve the pain of a sprained ankle. While the pack initially feels quite cold, it warms up after an hour of use. This is because energy in the form of heat is transferred to the ice pack from its warmer surroundings, including the athlete's ankle. Create a model that describes how much energy is gained by the pack over this time period. Describe what Input and Output terms may be present in the accounting equation.

Solution: For this model, the extensive property is energy. As shown in Figure 2.11, the system is the ice pack because the model is intended to capture the change in energy of the ice pack. The system boundary is the exterior of the ice pack. In this case, the extensive property of energy passes through the system boundary through *direct contact*. The primary Input term is the energy transferred from the ankle to the pack over the hour-long time period. Energy from the surrounding air also enters the system as an Input. Since the ice pack has a lower temperature than its surroundings, no energy as heat leaves the system as an Output term. ■

EXAMPLE 2.7 Wheelchair Frame Force Balance

Problem: A student is confined to a wheelchair after breaking his leg. As he is seated in the wheelchair, his body weight is supported by the wheelchair frame. Create a model of the patient in the wheelchair and determine the forces acting on the system. Describe what Input and Output terms may be present in the accounting equation.

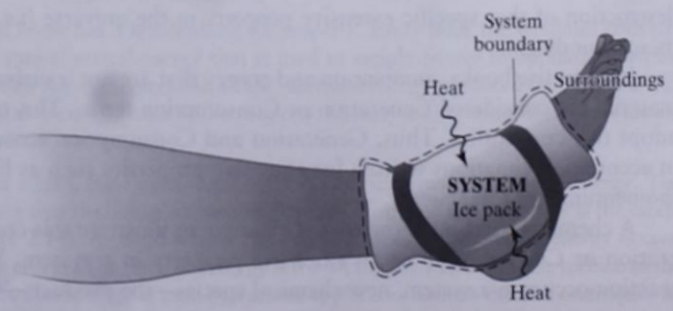
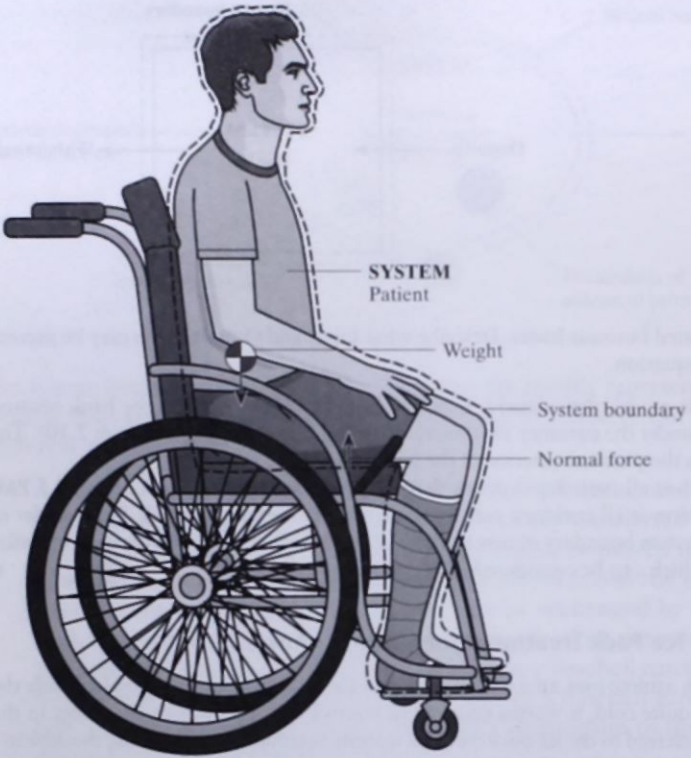


Figure 2.11
Energy as heat transferred to the ice pack.



on a wheelchair.

Solution: The system is the seated patient, and the boundary is the exterior of the patient's body. In this system, the patient's weight from the gravitational force is opposed by the upward, normal force from the surroundings, specifically the wheelchair, which exerts a normal force equal and opposite to the patient's weight (Figure 2.12). The gravitational force acting on the student in the wheelchair is an example of *nondirect contact*. These identified forces would be present in the Input and Output terms of a momentum conservation equation. ∎

2.4.2 Generation and Consumption Terms Describe Changes in the Universe

The **Generation** term describes the quantity of an extensive property that is created by the system, while the **Consumption** term describes the quantity that is used or destroyed within the system. The Generation and Consumption terms capture the production and elimination of an extensive property within the system. When a Generation or Consumption term is present, there is also a net production or destruction of that specific extensive property in the universe (i.e., the system plus its surroundings).

In some textbooks, momentum and energy that are not transferred through bulk material are considered Generation or Consumption terms. This textbook does not adopt this convention. Thus, Generation and Consumption terms are not present in accounting equations written for extensive properties such as linear and angular momentum and total energy.

A chemical reaction is the easiest example to illustrate this concept of the Generation or Consumption of an extensive property in a system. When a chemical reaction occurs in a system, new chemical species—the products—are formed as the

old chemical species—the reactants—are consumed. Consider the summary equation for photosynthesis:

$$6\ CO_2 + 6\ H_2O + \text{light} \rightarrow C_6H_{12}O_6 + 6\ O_2 \qquad [2.4\text{-}4]$$

In this case, the product chemical species—1 mole of glucose and 6 moles of oxygen—are counted as generated. The total amount of glucose and oxygen gas has increased in the system and the universe. On the other hand, the reactant chemical species—6 moles of carbon dioxide and 6 moles of water—have been consumed simultaneously, because the total amount of carbon dioxide and water has decreased in the system and in the universe. In summary, particular chemical species are generated and consumed in the photosynthesis reaction [2.4-4].

By balancing this equation, the number of elements (e.g., carbon, hydrogen, and oxygen) are equal on each side of the equation. Thus particular elements are neither generated nor consumed. As shown in Chapter 3, the *total mass* (products and reactants) as well as *mass* and *moles of individual elements* in the universe remain the same. In summary, mass and charge accounting equations may contain Generation and Consumption terms that stem from chemical reactions.

Conversion between different forms of energy such as mechanical energy and thermal energy, when considering the accounting equation for any given form of energy, is a second illustration of the Generation and Consumption terms in an accounting equation. In this text, electrical energy accounting equations (Chapter 5) and mechanical energy accounting equations (Chapter 6) contain Generation or Consumption terms that capture this conversion of energy from one form to another.

EXAMPLE 2.8 Daisy Photosynthesis and Carbon Dioxide Consumption

Problem: A daisy flower undergoes photosynthesis (equation [2.4-4]) in order to provide energy and sustenance for itself. Assume that CO_2 enters the system as a gas and that all entering CO_2 participates in the photosynthesis reaction. Consider the moles of CO_2 as the extensive property of interest and identify which terms in the accounting equation may be present. Also consider the moles of the element carbon.

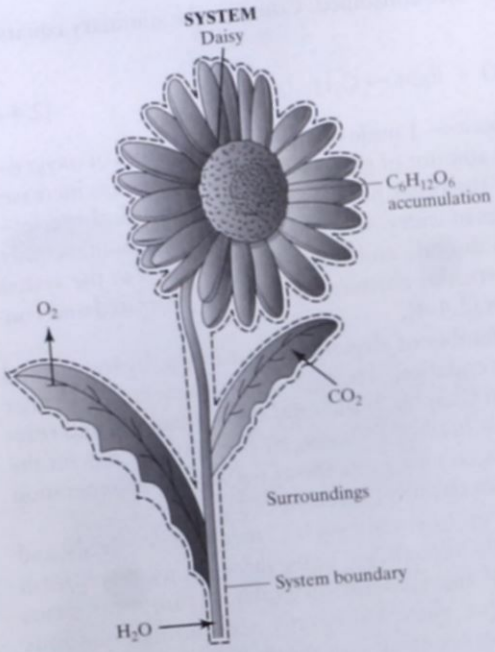
Solution: The extensive property is moles of CO_2. As shown in Figure 2.13, the system is the daisy and the boundary is the outside edge of the daisy. CO_2 enters the system by bulk transfer, therefore the Input term is present. CO_2 is consumed in the photosynthetic reaction, so there is a Consumption term. No CO_2 is generated through a chemical reaction, and no CO_2 leaves the system. There is no accumulation of CO_2.

Moles of carbon are a conserved property, so by definition the Generation and Consumption terms are not present. Moles of carbon enter the system (the daisy) as CO_2 and accumulate in the system as glucose. ■

EXAMPLE 2.9 Windmill Energy Generation

Problem: A small home has a windmill on its property. Every hour the windmill converts mechanical energy into electrical energy that is used to supply power to appliances within the home. Assume that the appliances use all the electrical energy produced by the windmill. Consider the electrical energy balance of the property. Identify which terms of the accounting equation may be present.

Solution: For this model, the extensive property is electrical energy, a type of energy. The system is the property that the house and windmill are built on, and the boundary is the estate property line (Figure 2.14). As wind rotates the windmill blades, mechanical energy is converted into electrical energy. The electrical energy is transferred by a line from the turbine to the house, where the electrical energy is used by various appliances such as the air conditioner or



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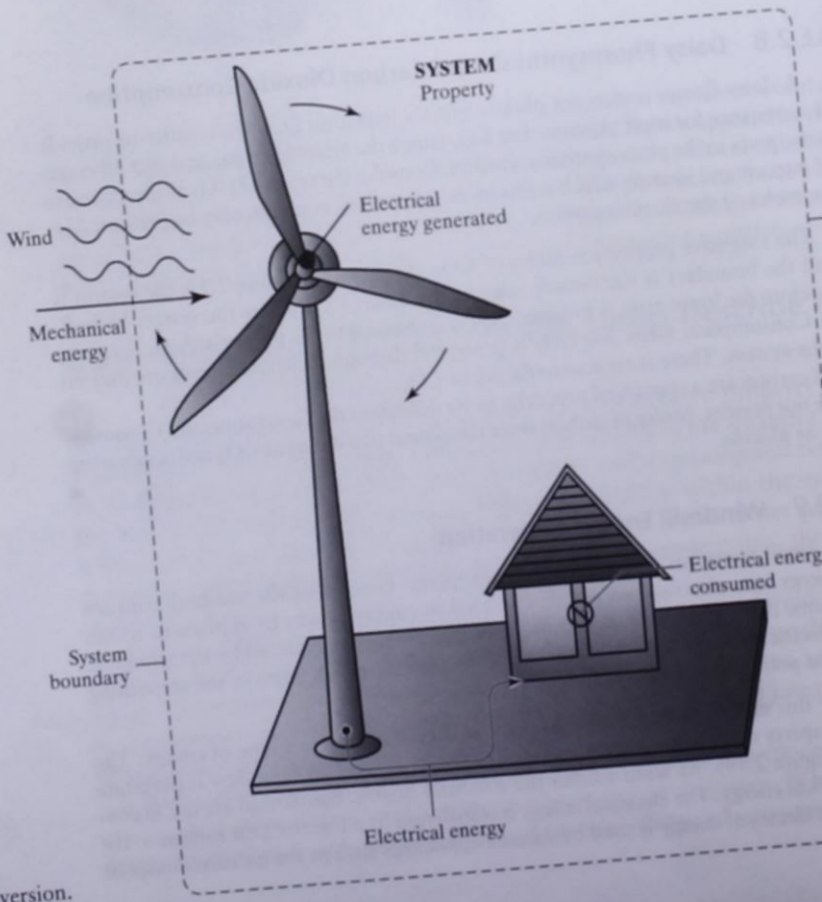


ure 2.14
dmill energy conversion.

refrigerator. No electrical energy crosses the system boundary, so neither the Input or Output terms are present. Instead, electrical energy is generated from mechanical energy harnessed by the windmill. All available electrical energy is used by the appliances in the house, and none leaves the system. Therefore, the Generation and Consumption terms are present. There is no accumulation of electrical energy in the system. ■

2.4.3 The Accumulation Term Describes Changes to the System

The **Accumulation** term describes the quantity that has collected in, or depleted from the system within the system boundary during the defined time period. In other words, the Accumulation term quantifies the net gain or loss of an extensive property within a system. A positive Accumulation term shows the increase of an extensive property in the system, whereas a negative Accumulation term shows the decrease of an extensive property in the system.

As an example, consider a water bottle you are taking to a baseball game. Prior to the game, you fill the bottle up with 1 L of water (Figure 2.15). While filling, the system (i.e., the water bottle) has a positive accumulation of 1 L of water. Over the course of the baseball game, you drink the entire contents of the water bottle. During the baseball game, the system has a negative accumulation of 1 L of water (Figure 2.15). In these cases, water moves in or out of the system and that movement results in a positive or negative accumulation of water.

However, the Accumulation term can be present or absent regardless of the movement of an extensive property. Determining whether a system has an Accumulation term or not depends entirely on whether an extensive property has been collected in or depleted from a system.

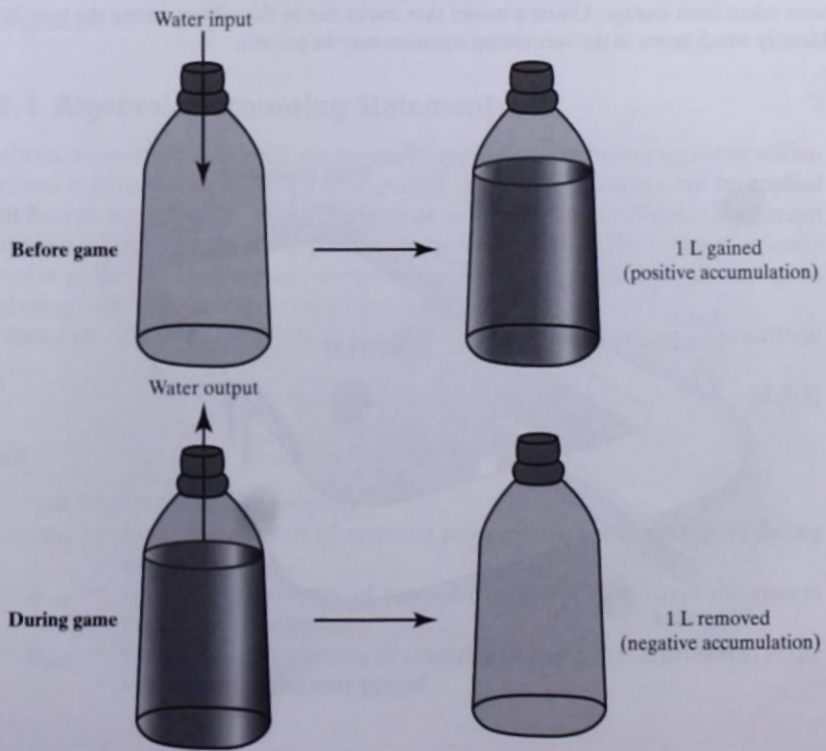


Figure 2.15
Accumulation in a water bottle.

The Accumulation term can be determined by finding the difference in quantity present in the system between the initial condition and the final condition, which are used to define the time interval during which the system is examined. The **initial condition** of a system is the state at the start of the interval, and the **final condition** of a system is the state at the end. The Accumulation term may also capture the rate at which an extensive property accumulates in a system.

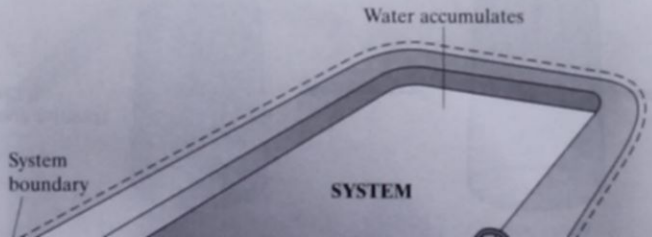
EXAMPLE 2.10 Filling a Pool

Problem: This summer, a neighbor down the street decides to build a swimming pool in their backyard. After constructing the basin, the pool is filled with copious amounts of water. Create a model that tracks the accumulation of the mass of water in the pool as it is initially filled. Identify which terms in the accounting equation may be present.

Solution: For this model, the extensive property is mass of water. The system for this problem is the swimming pool shown in Figure 2.16, and its boundary is the cement that lines the basin. The pool is selected as the system because the model should capture changes to water in the pool itself. During filling, water enters the system and is an Input term in the accounting equation. Water should not be undergoing any chemical reactions, so the Generation and Consumption terms are zero. An Output term could be present if the pool basin leaks water to the ground or if the water evaporates. There is a positive gain of water in the pool, hence there is a positive Accumulation term in the accounting equation. While filling up the pool, the rate of mass inflow of water approximately equals the rate of accumulation of water. ∎

EXAMPLE 2.11 Village Food Supply

Problem: A small village in Northern India farms in a self-sustaining manner, meaning that the village is wholly responsible for generating all the food it requires. One important crop is rice. In some years, the rice harvest is bountiful and an excess amount of rice is collected and then stored for later years that have lower rice production. Assume that 500 kg of rice are in storage at the beginning of 2015. In 2015, the crop yield for rice was low, so 300 kg were taken from storage. Create a model that tracks rice in this village during the year 2015. Identify which terms in the accounting equation may be present.



Solution: For this model, the extensive property is the mass of rice. The system is the Indian village, and its boundary is the edge of the village. Since the village is self-sustaining, no additional rice comes into or out of the village. As a result, there are no Input or Output terms in the accounting equation. The rice crop that is harvested is the Generation term in the accounting equation. The rice eaten by the village inhabitants is the Consumption term. We know that the Consumption term is larger than the Generation term because the rice reserves were used. The Accumulation term captures the difference in mass of rice from the initial to the final time. Initially, there are 500 kg of rice stored in the village, but at the end of 2015, there were only 200 kg of rice. Since people in the village ate 300 kg of reserved rice in 2015, there is a negative Accumulation term in the accounting equation. ■

2.5 Mathematical Framework for Accounting Equations

The **accounting equation** is a mathematical description of the movement, generation, consumption, and accumulation of an extensive property in a system of interest. Accounting equations can be written for extensive properties and the rates of those extensive properties that can be counted. The accounting equation is applied to the extensive properties stated in Section 2.4.

Accounting equations can be represented in three forms:

- Algebraic
- Differential
- Integral

The derivation of the differential and integral equations is covered in other textbooks (e.g., Bird, Stewart, and Lightfoot, *Transport Phenomena*, 2002). For this textbook, it is important that you understand all three equations and how to apply them appropriately. You should also be able to recognize that each equation is dimensionally homogenous.

2.5.1 Algebraic Accounting Statements

Algebraic accounting equations are generally applied to extensive properties within a defined system during a defined time period. **Algebraic equations can be applied when discrete quantities or "chunks" of extensive property are involved.** They cannot be applied when rates or time-dependent terms are involved. Examples of discrete extensive properties are a person, the momentum of a car, a pancreas, the energy in a lightning bolt, and the charge on a capacitor.

Based on equation [2.4-1], the generic algebraic accounting equation is written:

$$\psi_{in} - \psi_{out} + \psi_{gen} - \psi_{cons} = \psi_{acc}^{sys} \qquad [2.5\text{-}1]$$

where

ψ = Any extensive property
ψ_{in} = Input, the quantity of extensive property that enters the system during the time period
ψ_{out} = Output, the quantity of extensive property that leaves the system during the time period
ψ_{gen} = Generation, the quantity of extensive property that is generated in the system during the time period

ψ_{cons} = Consumption, the quantity of extensive property that is consumed in the system during the time period

ψ_{acc}^{sys} = Accumulation, the difference between the quantities of extensive property contained within the system at the end of the time period as compared to those at the beginning

The initial and final conditions can also be used to define Accumulation:

$$\psi_f - \psi_0 = \psi_{acc}^{sys} \qquad [2.5\text{-}2]$$

where

ψ_f = Final condition, the quantity of extensive property contained in the system at the end of the specified time period, and

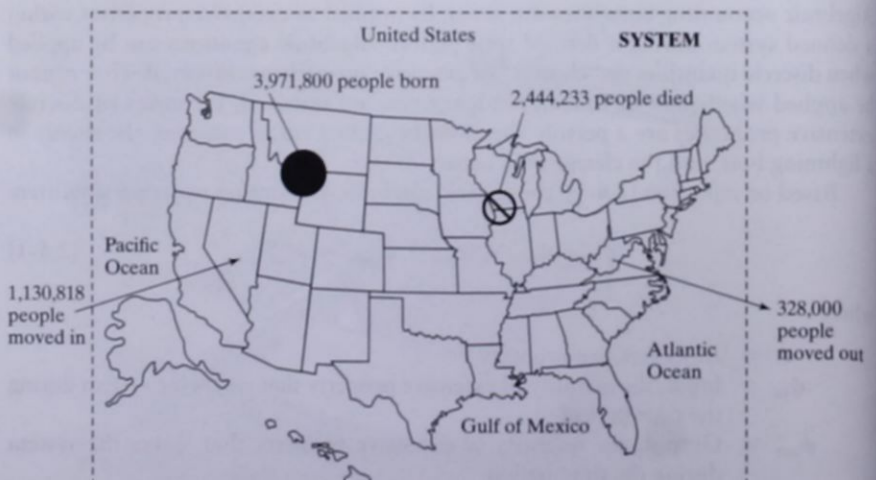
ψ_0 = Initial condition, the quantity of extensive property contained in the system at the beginning of the specified time period.

The dimension of the terms in equations [2.5-1] and [2.5-2] is that of the extensive property.

EXAMPLE 2.12 Population of the United States

Problem: Population growth has slowed in the United States since the 2008 economic recession, but the country continues to grow. Between January 1, 2009, and January 1, 2010, 1,130,818 people immigrated to the United States, 328,000 U.S. residents emigrated to other countries, 3,971,800 babies were born, and 2,444,233 people died (Figure 2.17).[1] Considering the population change in this year, would an algebraic, differential, or integral balance be most useful? Write a balance on the U.S. population during this time period to determine the net population change.

Solution: The property to be counted is people. The system is the United States, and the system boundary is defined by the national border and includes other entry or exit points using various transportation modes, such as airplanes. The time period is one year. Because a person can be considered a "chunk" of extensive property, an algebraic accounting equation is appropriate to use. The term ψ_{in} is 1,130,818 people, since this many people crossed



the system boundary (immigrated) into the United States; and ψ_{out} is 328,000 people, since this many people left the system across its boundary (emigrated). Because births increase the total number of people in the system and the universe, ψ_{gen} is 3,971,800 people; and because deaths decrease the total number of people in the system and in the universe, ψ_{cons} is 2,444,233 people. Therefore,

$$\psi_{acc}^{sys} = 1{,}130{,}818 \text{ people} - 328{,}000 \text{ people} + 3{,}971{,}800 \text{ people} - 2{,}444{,}233 \text{ people}$$

$$\psi_{acc}^{sys} = 2{,}330{,}385 \text{ people}$$

A net gain (i.e., positive accumulation) of 2,330,385 people occurred in the United States between January 1, 2009, and January 1, 2010. ■

EXAMPLE 2.13 Cell Culture

Problem: A research scientist maintains a cell culture of human dermal fibroblast (HDF) cells in a Petri dish. The scientist seeds 118,000 HDF cells into an empty Petri dish. Two days later, she or he removes all 150,000 HDF cells from the dish. If no HDF cells die, write a balance on the cell population during this period. Would an algebraic, differential, or integral balance be most useful? How many cells were produced by cell division during this period?

Solution: The property to be counted is HDF cells. The system is the Petri dish, and the system boundary is defined as the edge of the Petri dish. The time period is two days. Since cells can be considered a discrete amount and a fixed time period is specified, we select an algebraic statement to track the HDF cells. The term ψ_{in} is 118,000 cells, since this many cells enter the system boundary when the scientist seeds them into the empty Petri dish. The ψ_{cons} term is 0 cells since it states that no HDF cells die; in other words, no cells are eliminated or destroyed within the system and the universe. The term ψ_{out} is 150,000 cells, since this many cells are removed from the system after two days. Since there are no HDF cells in the Petri dish at the beginning of the time period and no HDF cells in the dish at the end of the time period, there is no accumulation of cells in the system. In other words, when the number of HDF cells at the initial and final conditions of the system are equivalent, there is no accumulation in the system. To calculate the number of cells generated:

$$\psi_{in} - \psi_{out} + \psi_{gen} - \psi_{cons} = \psi_{acc}^{sys}$$

$$118{,}000 \text{ cells} - 150{,}000 \text{ cells} + \psi_{gen} - 0 = 0$$

$$\psi_{gen} = 32{,}000 \text{ cells}$$

32,000 HDF cells are generated during the two-day time period. This indicates that approximately 27% of the cells divided, increasing the number of cells in the system and the universe. ■

2.5.2 Differential Accounting Statements

Rate is defined as the amount an extensive property changes per unit time, t:

$$\text{rate} = \frac{\Delta EP}{\Delta t} \qquad [2.5\text{-}3]$$

where ΔEP is the change in the extensive property and Δt is the change in time. The units of these rates are extensive property divided by time. Rate is denoted with a dot over the variable.

Recall the list of extensive properties introduced at the outset of Section 2.4. All of these extensive properties can be defined as a "chunk" or discrete quantity, as well as a rate of extensive property. Rates of an extensive property are used frequently in this textbook. Some rates are specified as flow rates. A **flow rate** describes the transport of an extensive property over a period of time. A flow rate can also be

described as the amount of an extensive property that flows by a point over time. A conceptual example of mass flow rate is the flow of water from a garden hose. A specific example is the mass flow rate of exhalation of air, at 6 g/min. An example of charge flow rate is current passing through a wire in a circuit. An example of heat flow is the rate of energy transferred down a temperature gradient. Note that each rate given is an extensive property divided by time.

A flow rate can transport an extensive property of interest into and out of a system over time by inlet and outlet streams. A **stream** is the pathway by which an extensive property enters or leaves a system. Streams transferring an extensive property through the transfer of mass are labeled as straight line arrows in diagrams in this book. Streams transferring an extensive property but without mass (e.g., work) are labeled as curvy arrows in this book.

In addition to transfer across a system boundary, an extensive property can be generated, consumed, or accumulated at a specified rate in a system. Thus, in the context of accounting equations, rate can also be used to describe the change of an extensive property over time as captured by the Generation, Consumption, and Accumulation terms. An example of consumption in a biological reaction is the rate of oxygen metabolism in tissues at 0.64 mg/s.

The differential form of the accounting statement is most appropriate when extensive properties are specified as rates. The differential form of the accounting equation is written:

$$\dot{\psi}_{in} - \dot{\psi}_{out} + \dot{\psi}_{gen} - \dot{\psi}_{cons} = \dot{\psi}_{acc}^{sys} = \frac{d\psi}{dt} \qquad [2.5\text{-}4]$$

where

$\dot{\psi}_{in}$ = rate at which an extensive property enters the system,

$\dot{\psi}_{out}$ = rate at which an extensive property leaves the system,

$\dot{\psi}_{gen}$ = rate at which an extensive property is generated in the system,

$\dot{\psi}_{cons}$ = rate at which an extensive property is consumed in the system, and

$\dot{\psi}_{acc}^{sys}$ or $\frac{d\psi}{dt}$ = rate at which an extensive property accumulates within the system.

Note that $\dot{\psi}_{in}$, $\dot{\psi}_{out}$, $\dot{\psi}_{gen}$, $\dot{\psi}_{cons}$, and $\dot{\psi}_{acc}^{sys}$ are all rates. The accumulation term, $d\psi/dt$, is often expressed as the instantaneous rate of change of the extensive property in the system.

The dot over the symbol ψ signifies a rate, a change in the extensive property with respect to time, which is the mathematical definition of a derivative. For example, if ψ is defined as the extensive property mass, then $\dot{\psi}$ is the change of mass in the system over time or the mass flow rate. If ψ is defined as charge, then $\dot{\psi}$ is the time derivative of charge or the current. The dimension of the terms in equation [2.5-4] is that of the extensive property divided by time. Table 2.2 lists the most commonly used extensive properties as amounts and rates.

Equation [2.5-4] is written for a differential time period (dt); hence, this form of the accounting statement is called a differential balance. It can be constructive to think about the differential accounting equation from two perspectives. At the macroscopic level, a differential equation is often used when the system is operating on an ongoing or continuous basis. These systems can be seen as going "forever" or without a specified end in mind. In this case, the rates or streams are seen as macroscopic rates, with extensive property moving, being generated or consumed, and/or

TABLE 2.2

Extensive Properties and Their Associated Rates

Extensive Property	Variable	Rate of Extensive Property	Variable
Mass	m	Rate of mass	$\dot{m}$
Mass of individual species	m_s	Rate of mass of individual species	$\dot{m}_s$
Mass of individual element	m_p	Rate of mass of individual element	$\dot{m}_p$
Moles	n	Rate of moles	$\dot{n}$
Moles of individual species	n_s	Rate of moles of individual species	$\dot{n}_s$
Moles of individual element	n_p	Rate of moles of individual element	$\dot{n}_p$
Total energy	E_T	Rate of total energy	$\dot{E}_T$
Internal energy	U	Rate of internal energy	$\dot{U}$
Electrical energy	E_E	Rate of electrical energy	$\dot{E}_E$
Enthalpy	H	Rate of enthalpy	$\dot{H}$
Net electrical charge	q	Rate of charge or current	$\dot{q}$
Linear momentum	$\vec{p}$	Rate of linear momentum	$\dot{\vec{p}}$
Angular momentum	$\vec{L}$	Rate of angular momentum	$\dot{\vec{L}}$

accumulating in the system. On the other hand, at the instantaneous level, you can think of the differential accounting statement as describing what is happening in the system at an instant in time. To solve a differential equation in general, a boundary or initial condition must be specified. Depending on the problem, the dependent variable, ψ, may be specified at some value of the independent variable (in this case, time). Frequently, the initial condition of the system at $t = 0$ is specified.

EXAMPLE 2.14 Power Dissipated Across a Resistor

Problem: A set of two resistors with unequal impedance, or resistance, are placed in series with a power supply, creating a closed circuit. Once started, one resistor consumes 33 joules of electrical energy every minute and the other consumes 21 joules every minute. If the components use up all the available energy, what is the power rating, or joules produced per second, of the supply? To determine the power rating, would an algebraic, differential or integral balance be most useful?

Solution: The extensive property is rate of electrical energy. The system is the circuit. The system boundary is the edge of the wires connecting the resistors and power supply. A differential conservation statement is selected because rates of electrical energy are given and the time period is ongoing.

In this problem, electrical energy is being produced by the power supply and consumed by the two components. There is no electrical energy leaving or entering the system across the system boundary, thus the accounting equation is:

$$\dot{\psi}_{gen} - \dot{\psi}_{con} = \dot{\psi}^{sys}_{acc}$$

Since the components use up all available energy, there is no accumulation in the system. To find the total electrical energy generated in the system, we substitute the two consumption terms of 33 J/min and 21 J/min:

$$\dot{\psi}_{gen} - \left(33\frac{J}{min}\right)\left(\frac{1\ min}{60\ s}\right) - \left(21\frac{J}{min}\right)\left(\frac{1\ min}{60\ s}\right) = 0$$

$$\dot{\psi}_{gen} = 0.9\ W$$

Thus, the power supply is rated for 0.9 W. ■

EXAMPLE 2.15 Oil Consumption in the United States

Problem: On average, the United States consumes oil at a rate of 19 million barrels/day (Figure 2.18). Yet, the United States produces only 8.7 million barrels/day. What is the mass of oil that the United States must import in order to account for this deficit? To determine this, would an algebraic, differential, or integral statement be most useful? A barrel holds 42 gallons and the density of oil is 825 kg/m^3.

Solution: The extensive property to be measured is the rate of oil. The system of interest is the United States, and its boundary is the national border and includes any entry and egress point for oil imports. Accounting and conservation equations should be written for mass, not volume. Compressible fluids (especially gases) may invalidate volume-based accounting equations. For that reason, it is always better to use mass accounting equations.

Since the extensive property is a rate, we will use a differential accounting statement. The 8.7 million barrels of oil produced and refined from the ground every day is captured in the $\dot{\psi}_{gen}$ term; the 19 million barrels of oil consumed per day is captured by the $\dot{\psi}_{cons}$ term:

$$\dot{\psi}_{gen} = 8.7 \times 10^6 \frac{\text{barrels}}{\text{day}}\left(\frac{42 \text{ gal}}{\text{barrel}}\right)\left(\frac{\text{m}^3}{264.17 \text{ gal}}\right)\left(\frac{825 \text{ kg}}{\text{m}^3}\right) = 1.1 \times 10^9 \frac{\text{kg}}{d}$$

$$\dot{\psi}_{gen} = 1.9 \times 10^7 \frac{\text{barrels}}{\text{day}}\left(\frac{42 \text{ gal}}{\text{barrel}}\right)\left(\frac{\text{m}^3}{264.17 \text{ gal}}\right)\left(\frac{825 \text{ kg}}{\text{m}^3}\right) = 2.5 \times 10^9 \frac{\text{kg}}{d}$$

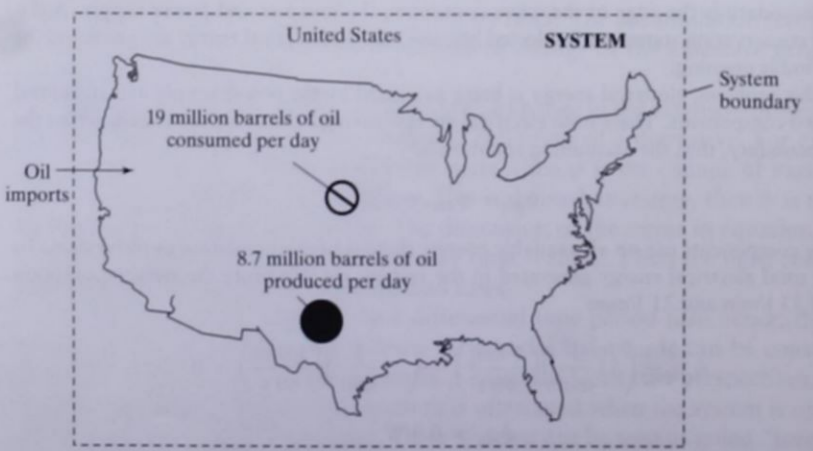
Lacking further information for the problem, we assume that exports are negligible and there is no accumulation of oil. Thus, $\dot{\psi}_{out}$ and $\dot{\psi}_{acc}$ are set to zero. The term $\dot{\psi}_{in}$ captures the mass rate of oil that needs to be imported.

$$\dot{\psi}_{in} - \dot{\psi}_{out} + \dot{\psi}_{gen} - \dot{\psi}_{cons} = \dot{\psi}_{acc}^{sys}$$

$$\dot{\psi}_{in} - 0 + 1.1 \times 10^9 \frac{\text{kg}}{\text{day}} - 2.5 \times 10^9 \frac{\text{kg}}{\text{day}} = 0$$

$$\dot{\psi}_{in} = 1.4 \times 10^9 \frac{\text{kg}}{\text{day}}$$

Thus, the United States imports about 1.4 billion kg of oil every day (approximately 10.4 million barrels of oil/day). ∎



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2.5.3 Integral Accounting Statements

Finally, the accounting statement can be written in an integral form. **Integral balances are most useful when trying to evaluate conditions between two discrete time points.** Integral accounting equations can be written to incorporate rates of change of an extensive property. When developing an integral balance, you can write the differential balance equation and integrate it between the initial and final times. The integral accounting statement is:

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt + \int_{t_0}^{t_f} \dot{\psi}_{\text{gen}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{cons}}\, dt = \int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}\, dt \quad [2.5\text{-}5]$$

where

$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt$ = total extensive property that enters the system between the initial time t_0 and final time t_f,

$\int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt$ = total extensive property that leaves the system between t_0 and t_f,

$\int_{t_0}^{t_f} \dot{\psi}_{\text{gen}}\, dt$ = total extensive property that is generated in the system between t_0 and t_f,

$\int_{t_0}^{t_f} \dot{\psi}_{\text{cons}}\, dt$ = total extensive property that is consumed in the system between t_0 and t_f, and

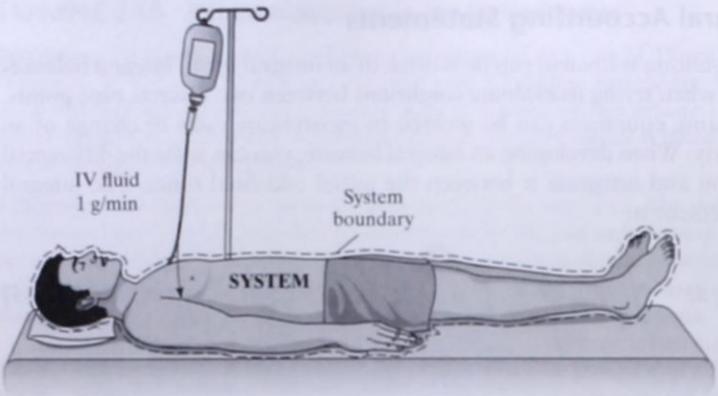
$\int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}\, dt$ = total extensive property that accumulates in the system between t_0 and t_f.

The Accumulation term can also be written as $\int_{t_0}^{t_f} \frac{d\psi}{dt} dt$ or $\int_{\psi_0}^{\psi_f} d\psi$ where ψ_f and ψ_0 are values of the extensive property at the final and initial conditions, respectively.

Selecting the form of the Accumulation term in the integral accounting and conservation equations can be difficult. The $\int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}$ form can be appropriate when the rate of accumulation within a system is given. For example, consider the system as a man who is receiving an IV, as depicted in Figure 2.19. If he is accumulating IV fluid at a rate of 1 g/min for 1 hour, then it makes sense to write the Accumulation term as: $\int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}} = \int_{0}^{1\text{ hour}} 1 \frac{\text{g}}{\text{min}} dt.$

In cases where $\dot{\psi}_{\text{acc}}^{\text{sys}}$ is not a function of time, $\int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}$ can be integrated to $\dot{\psi}_{\text{acc}}^{\text{sys}} (t_f - t_0)$. In cases when $\dot{\psi}_{\text{acc}}^{\text{sys}}$ is a function of time, additional mathematical steps will be needed.

The $\int_{t_0}^{t_f} \frac{d\psi}{dt} dt$ form of the Accumulation term can be appropriate when $d\psi/dt$ is a function of time. When $d\psi/dt$ is specified as a function of time, it can often be integrated between two time points.



ing fluid from

The $\int_{\psi_0}^{\psi_f} d\psi$ form of the Accumulation term can be appropriate when the values of the extensive property at the initial and final times are known. When $d\psi$ can be integrated, the term can be written as $\psi_f - \psi_0$, or the difference in extensive property in the system during the specified time period. As an alternate, consider the system as the IV bag that delivered fluid to the man. If the value of ψ_f is 670 g and the value of ψ_0 is 730 g, then the Accumulation term is calculated as the difference between the final and initial mass valves, or −60 g.

Like the terms in differential accounting equations, $\dot{\psi}_{in}$, $\dot{\psi}_{out}$, $\dot{\psi}_{gen}$, $\dot{\psi}_{cons}$, and $\dot{\psi}_{acc}^{sys}$ are rates. All $\dot{\psi}$ terms can be specified as a function of time. The dimension of the terms in equation [2.5-5] is that of the extensive property. Information on the conditions of the system at either t_0 or t_f or both is usually needed to solve problems using the integral equation. For simple systems whose terms do not change with time, the integral equation [2.5-5] can be reduced to the algebraic equation [2.5-1].

The differential and integral forms are particularly useful, since rates are often given to specify the operation of bioengineering systems. The characteristics of the algebraic, differential, and integral accounting equations are summarized in Table 2.3.

EXAMPLE 2.16 Diabetes Medication

Problem: Jean is a type II diabetic, which means her cells are insensitive to insulin, a hormone that helps cells take in glucose. Her physician has prescribed one 30-mg Pioglitazone pill a day to promote the uptake of glucose into muscle cells. Her body metabolizes the drug at a rate dependent on the time since she ingested the pill:

$$\text{rate of drug metabolism} = ae^{-kt}$$

stics of Accounting Equations

	Algebraic	Differential	Integral
orate discrete quantity	Yes	No	Sometimes
e property?			
l	Finite	Ongoing or Instantaneous	Finite
orate rates?	No	Yes	Yes
f equation	Extensive property	Extensive property	Extensive property

where a is a rate constant equal to 45 mg/day and k is an unknown rate constant. At the end of the 24-hour dosage period, Jean's physician finds that 2 mg of unmetabolized drug remains in her bloodstream. Assume that the rest of the drug is completely metabolized and no Pioglitazone is created or leaves her body through other means (e.g., urine). Is it best to use an algebraic, differential, or integral accounting equation to calculate k? Why? Consider Jean's body as the system. Calculate the metabolic rate constant k for Pioglitazone.

Solution: The drug consumption is given as a rate. Since the drug Pioglitazone is converted into another chemical, the net amount of Pioglitazone in the universe has decreased. Thus, the drug is not conserved in this problem. The rate of metabolism should be considered a Consumption term. Since the consumption rate is time dependent and a finite period of time (one day) is given, the integral accounting equation [2.5-5] is most appropriate.

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt + \int_{t_0}^{t_f} \dot{\psi}_{\text{gen}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{cons}}\, dt = \int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}\, dt$$

If we define that the one-day time period starts immediately after Jean ingests the pill, the Input term is zero. Because no drug leaves her body unmetabolized, the Output term is zero. No drug is generated in Jean's body, so the Generation term is eliminated. Thus, the above equation reduces to:

$$-\int_{t_0}^{t_f} \dot{\psi}_{\text{cons}}\, dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

where $\dot{\psi}_{\text{cons}}$ is the rate of drug consumption (ae^{-kt}), ψ_0 is the amount of drug present in Jean's body at the beginning of the time period (30 mg), and ψ_f is the amount of drug remaining in her system at the end of the time period (2 mg). Substituting the known values and integrating from $t_0 = 0$ to $t_f = 1$ day, we obtain:

$$-\int_0^{1\text{ day}} ae^{-kt}\, dt = 2\text{ mg} - 30\text{ mg}$$

$$\frac{a}{k}e^{-kt}\Big|_0^{1\text{ day}} = \frac{a}{k}e^{-k(1\text{ day})} - \frac{a}{k} = -28\text{ mg}$$

Substituting in 45 mg/day for a and solving the equation for k in a computer software program such as MATLAB or Excel yields $k = 1.04\text{ day}^{-1}$. Therefore, the rate constant associated with Jean's metabolism of Pioglitazone is $k = 1.04\text{ day}^{-1}$. ■

had limited success. Until a cure—possibly one involving tissue engineering—is found, Banting and Best's revolutionary medical discovery of insulin remains the best treatment for diabetic patients.

Diabetes and its treatment provide an example of physiological constraints on an accounting equation. Insulin is consumed during normal metabolism. However, for a person with Type I diabetes, there is no Generation term. Consequently, the insulin Consumption term must be balanced by regular injections of insulin (an Input term) or the implantation of islets of Langerhans (a Generation term).

EXAMPLE 2.17 Red Blood Cell Replenishment

Problem: You donate a pint of whole blood at a blood drive. After donating, an assistant tells you to not donate again for at least eight weeks. The assistant claims that this time period is sufficient to ensure that your body replenishes the amount of blood you lost in the donation.

As a biomedical engineer, you know that 1 mL of blood contains approximately 5×10^6 red blood cells (RBCs). Human bone marrow generates an average of 2×10^{11} RBCs every day.[3] Typically, the generation of RBCs is balanced by degradation (i.e., death) and clearance of RBCs, so that the total number of RBCs in an adult is a constant.

After donation, the body's bone marrow generates more RBCs to replenish lost RBCs. When reduced blood volume is detected, the hormone erythropoietin is released, thereby increasing RBC production. You estimate that the released erythropoietin increases RBC generation by 30%. Is it best to use an algebraic, differential, or integral accounting equation? Estimate if eight weeks is an adequate time period to replenish donated RBCs.

Solution: The property to be counted is rate of RBCs. The system is the human body, and the system boundary is the skin. An integral accounting equation is selected because the system involves rates (e.g., RBC generation rates) and a finite time period (even though the length of the time period itself is unknown).

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt + \int_{t_0}^{t_f} \dot{\psi}_{\text{gen}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{con}}\, dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

The form of the Accumulation term $\int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$ was selected because the difference in RBC contained in the system over the time of interest is known.

The number of RBCs lost through the whole blood donation is calculated as:

$$1 \text{ pint} \left(\frac{1 \text{ qt}}{2 \text{ pints}}\right)\left(\frac{10^6 \text{ mL}}{1056.68 \text{ qt}}\right)\left(\frac{5 \times 10^9 \text{ RBC}}{\text{mL}}\right) = 2.37 \times 10^{12} \text{ RBC}$$

To solve this problem, it is not necessary to know the total number of RBCs in the system of the body. Rather, the difference in the total RBCs between the final and initial time is the amount accumulated:

$$\psi_f - \psi_0 = 2.37 \times 10^{12} \text{ RBC}$$

No RBCs cross the system boundary, or enter or leave the system; thus the Input and Output terms equal zero. The Generation term will be estimated as 130% of the normal generation rate, since we are estimating a 30% increase in RBC generation. We assume that the consumption (i.e., degradation or death) of RBCs equals the normal generation rate:

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{gen}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{con}}\, dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

$$\int_{t_0}^{t_f} (1.3)\left(2 \times 10^{11} \frac{\text{RBC}}{\text{day}}\right) dt - \int_{t_0}^{t_f} \left(2 \times 10^{11} \frac{\text{RBC}}{\text{day}}\right) dt = 2.37 \times 10^{12} \text{ RBC}$$

When $t_0 = 0$, the equation is integrated with respect to time:

$$\left(2.6 \times 10^{11} \frac{\text{RBC}}{\text{day}}\right) t_f - \left(2 \times 10^{11} \frac{\text{RBC}}{\text{day}}\right) t_f = 2.37 \times 10^{12}\ \text{RBC}$$

$$t_f = 39.5\ \text{days} \cong 40\ \text{days}$$

Therefore, it takes approximately 40 days (almost six weeks) for your body to replenish the RBCs lost from a donation. This demonstrates that eight weeks is enough time for your body to recover from a single pint donation. ■

2.6 Mathematical Framework for Conservation Equations

The **conservation equation** is a mathematical description of the movement and accumulation of an extensive property in a system of interest. As compared to the accounting equation, the conservation equation does not have the Generation or Consumption terms. The conservation equation can only be applied to a subset of the extensive properties listed for use in an accounting equation, specifically the following:

- Total mass
- Mass of individual element
- Moles of individual element
- Total energy
- Net charge
- Linear momentum
- Angular momentum

Like accounting equations, conservation equations can be represented in three forms:

- Algebraic
- Differential
- Integral

A good rule of thumb when approaching a problem requiring an accounting or conservation equation is to start with the accounting equation. Unnecessary terms will fall out of the equation as the problem is solved. However, for known conserved properties, it is appropriate to begin with a conservation equation.

2.6.1 Algebraic Conservation Equation

To write an algebraic conservation equation, we rewrite equation [2.5-1] as follows:

$$\psi_{\text{in}} - \psi_{\text{out}} + \psi_{\text{gen}} - \psi_{\text{cons}} = \psi_{\text{acc}}^{\text{sys}} \qquad [2.6\text{-}1]$$

As a conserved extensive property is neither created nor destroyed, the algebraic accounting equation is reduced by setting ψ_{gen} and ψ_{cons} equal to zero.

$$\psi_{\text{in}} - \psi_{\text{out}} = \psi_{\text{acc}}^{\text{sys}} \qquad [2.6\text{-}2]$$

where ψ is any extensive property, ψ_{in} is the Input of that property, ψ_{out} is the Output, and $\psi_{\text{acc}}^{\text{sys}}$ is the Accumulation during the defined time period. (Refer to

the algebraic accounting equation in Section 2.5.1 for more complete definitions of these variables.)

The initial and final conditions can also be used to define the Accumulation term:

$$\psi_f - \psi_0 = \psi_{acc}^{sys} \quad [2.6\text{-}3]$$

where ψ_f is the quantity of property of the system present at the final condition, and ψ_0 is the quantity present at the initial condition.

EXAMPLE 2.18 Cooling Tap Water

Problem: A glass of tap water is left on a counter. Before drinking the contents of the glass, you decide to lower the temperature of the water by adding six reuseable ice cubes. Five minutes later, the glass feels colder. Is it best to use an algebraic, differential, or integral accounting equation to describe the system of the water in the glass? Write a balance to describe the total energy transferred between the water and the ice.

Solution: The extensive property is energy. The system is the water (not the reusable ice cubes) in the glass, and the system boundary is the glass itself. The question is asking to describe the total energy, which is a discrete quantity. Given this and a fixed time period, an algebraic accounting equation is appropriate. As has been previously noted, total energy is conserved, meaning it can be neither created nor destroyed, thus a conservation equation is appropriate:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys}$$

With ice cubes in the system, energy is transferred as heat from the warmer water to the colder ice cubes. This process of heat transfer is elaborated further in Chapter 4. Note that this heat transfer is an example of direct contact (not bulk movement).

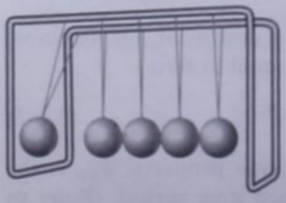
No other forms of energy, such as potential energy, kinetic energy, or work, are transferring into or out of the system. Therefore, the conservation equation can be reduced to:

$$-\psi_{out} = \psi_{acc}^{sys}$$
$$-Q = \psi_{acc}^{sys} = E_f^{sys} - E_0^{sys}$$

where Q is the net heat that transferred from the water to the ice in 5 min. Since the system is losing energy as heat, the final energy of the system is lower than the initial energy. This difference in energy of the water is physically manifested by the lower temperature of the water. ∎

EXAMPLE 2.19 Momentum in a Newton's Cradle

Problem: A Newton's Cradle is comprised five identically sized metal balls that are each suspended from a frame by two wires of equal length angled away from each other (Figure 2.20). This restricts the movement of the balls to mimic that of a series of pendulums. You decide to analyze the transfer of momentum of an idealized Newton's Cradle. You pull the first ball away from the set of five and let it drop. As it strikes the other four, the first ball comes to a complete stop as it transfers its momentum to the ball at the far end of the cradle, which swings upward. Is it best to use an algebraic, differential, or integral accounting



is conserved in a
adle.

equation? Write a momentum balance on the Newton's Cradle that tracks the conservation of momentum during the transfer of momentum from the first to the fifth ball during a single cycle.

Solution: In this problem, the extensive property is total momentum. The system is the set of three balls that are in the center of the cradle, and the boundary is the edge of the second and fourth balls. The system boundary must be specially defined in this case in order to demonstrate how momentum passes through the system and is transferred from the first ball to the fifth ball. This problem requires tracking of a discrete amount of momentum over a defined time period, thus an algebraic statement is appropriate. Since total momentum is a conserved property, we use the conservation equation:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys}$$

In this case, the input momentum $\vec{p}_{in,\,ball1}$ comes from the impact of the first ball striking our system. Since none of the middle three balls move when they are impacted, there is no accumulation of momentum in the system. The output momentum $\vec{p}_{out,\,ball5}$ is thus equal to the input momentum:

$$\psi_{in} - \psi_{out} = 0$$
$$\vec{p}_{in,\,ball1} = \vec{p}_{out,\,ball5}$$

■

EXAMPLE 2.20 Ejection Fraction

Problem: Ejection fraction (E_F) is defined as the fraction of blood expelled from a ventricle during one heart contraction, or heartbeat. Left ventricular ejection fraction typically ranges from 55% to 75%. After contraction in a healthy heart, the percentage of blood that is not expelled remains in the ventricle.

In a heart with a low E_F, excess blood can flow back through the atrium and cause a pressure imbalance between the heart and pulmonary vasculature. This pressure imbalance can lead to fluid accumulation in the lungs that can impede breathing.

An elderly patient comes in for a heart echocardiogram. During the test, the physician measures that the left ventricle pumps out 70 g of blood in a single contraction. After further calculation, the physician determines that the patient's left ventricular E_F is 65%, within the normal range. Assuming an E_F of 65%, calculate how much blood remains in the patient's left ventricle after a single heartbeat. Is it best to use an algebraic, differential, or integral accounting equation to describe this system?

Solution: The extensive property being tracked is the mass of blood. The system is the left ventricle, and the system boundary is the ventricular wall (including the mitral valve and the aortic valve). An algebraic conservation equation is most suitable for this problem since discrete quantities of blood, rather than rates, are being tracked; in addition, there is a discrete time period of one contraction.

There is no generation or consumption of blood, as it is not involved in any reactions. Thus, the accounting equation simplifies to a conservation equation:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys}$$

The ejection fraction can be written as the mass of blood leaving the left ventricle (ψ_{out}) divided by the mass that comes in (ψ_{in}). Substituting E_F and ψ_{out}, ψ_{in} can be be calculated:

$$E_f = \frac{\psi_{out}}{\psi_{in}}$$
$$0.65 = \frac{70\ \text{g}}{\psi_{in}}$$
$$\psi_{in} = 108\ \text{g}$$

To determine the amount of blood that remains in the ventricle after a contraction, the algebraic conservation equation is applied:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys}$$
$$108\ g - 70\ g = 38\ g = \psi_{acc}^{sys}$$

Thus, 38 g of blood accumulate in the ventricle after a single heartbeat.

2.6.2 Differential Conservation Equation

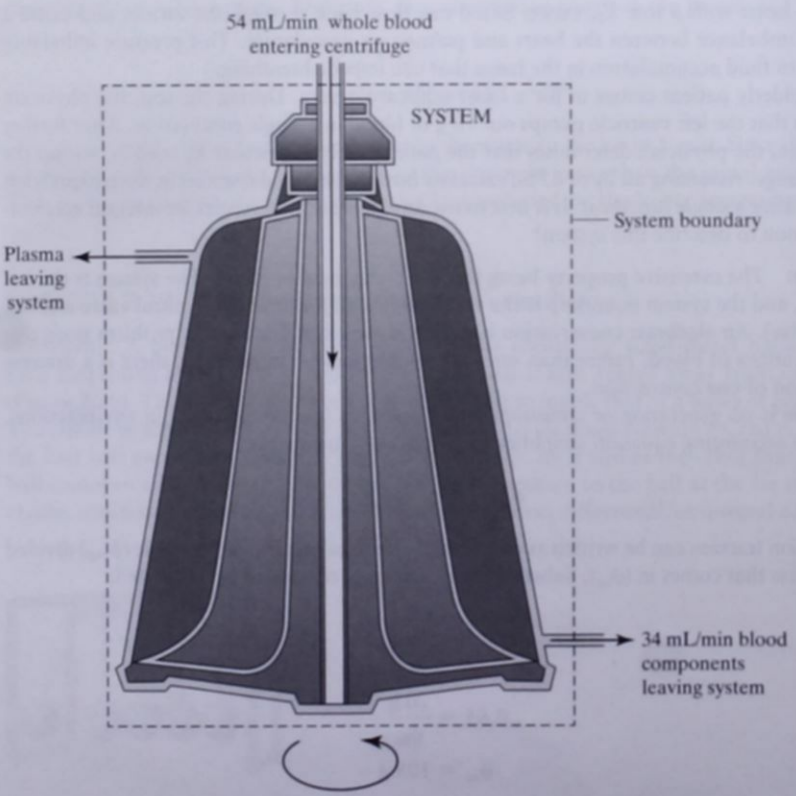
The conservation equation, just like the accounting equation, can be written in differential form:

$$\dot{\psi}_{in} - \dot{\psi}_{out} = \dot{\psi}_{acc}^{sys} = \frac{d\psi}{dt} \qquad [2.6\text{-}4]$$

where $\dot{\psi}_{in}$ is the rate of Input, $\dot{\psi}_{out}$ is the rate of Output, and $\dot{\psi}_{acc}$ or $d\psi/dt$ is the rate of Accumulation. For more complete definitions of these variables, refer to the differential accounting equation in Section 2.5.2.

EXAMPLE 2.21 Plasmapheresis Machine

Problem: Every minute, 54 mL of blood enter a plasmapheresis machine (see Section 1.6.4). After entering the machine, the blood travels to a centrifugation chamber where the blood plasma is separated from other blood components and pumped into a collecting tube (Figure 2.21). After this process, the remaining blood components (largely comprised of cells) are pumped out of the centrifugation chamber and into the blood exit tube where replacement



onents are sepa-
plasmapheresis

saline is added to make up for the lost plasma. If 34 mL/min of blood components leave the centrifugation chamber to be returned to the body, what is the rate of replacement saline addition? Is it best to use an algebraic, differential, or integral accounting equation? Assume that there is no change in the overall volume of blood in the patient and that saline has the same density as blood.

Solution: For this problem, the extensive property is the rate of blood mass. We consider the centrifuge within the plasmapheresis machine as our system and draw our boundary around just the centrifuge. The flow of blood into the system and the flows of blood components and plasma out of the system are shown with marked arrows. The placement of the system boundary is selected to cut across the flow rates of interest.

First we assume that all blood entering the plasmapheresis machine enters the centrifuge. Since mass conservation equations should be written for mass rate and not volume rate, we convert the volumetric flow rate of blood into mass flow rate of blood using the density of blood of 1.056 g/mL:

$$\dot{\psi}_{in} = 54\frac{mL}{min}\left(\frac{1.056\ g}{mL}\right) = 57\frac{g}{min}$$

For this example, a differential conservation statement is used since we are dealing with rates and no time period is specified. The centrifugation of blood is a physical, rather than chemical process. Thus, there is no generation or consumption of blood and a conservation equation is appropriate. Similarly, the volume rate of blood components (BC) is converted into a mass flow rate:

$$\dot{\psi}_{out,\,BC} = 34\frac{mL}{min}\left(\frac{1.056\ g}{mL}\right) = 35.9\frac{g}{min}$$

Because the system is ongoing and no blood or blood components accumulate in the system, the Accumulation term in the conservation equation is set to zero. The Output term is broken into two parts: the blood components and the plasma.

$$\dot{\psi}_{in} - \dot{\psi}_{out} = \dot{\psi}_{acc}^{sys} = 0$$

$$\dot{\psi}_{in} - \dot{\psi}_{out,\,BC} - \dot{\psi}_{out,\,plasma} = 0$$

$$\dot{\psi}_{out,\,plasma} = \dot{\psi}_{in} - \dot{\psi}_{out,\,BC}$$

$$\dot{\psi}_{out,\,plasma} = 57\frac{g}{min} - 35.9\frac{g}{min} = 21.1\frac{g}{min}$$

Since 21.1 g/min of plasma are removed, 21.1 g/min of replacement saline must be added to the blood components exit tube to keep the blood volume of the patient constant. The volumetric flow rate of saline components calculated using the stated density:

$$21.1\frac{g}{min}\left(\frac{1\ mL}{1.056\ g}\right) = 20\frac{mL}{min}$$

■

EXAMPLE 2.22 Bank Account

Problem: On the first day of every month, you receive a bank statement for your checking account that lists the previous month's activity and your current balance. Transactions for each month of the year remain the same: You collect $5 of interest and spend $75 on books, $150 on food, $40 on your phone bill, $50 on utilities, and $400 on rent. Fortunately, you have a job that pays you $450 every two weeks, and you are very diligent about depositing all of your money into your account. To calculate your savings rate, would an algebraic, differential, or integral balance be most appropriate? Write a balance on the money in your bank account. Assume that there are four weeks in a month.

Solution: The property to be counted is money. The system is your bank account, and the time period is ongoing. Money goes into and out of your account each month at a certain rate. Therefore, the differential conservation equation is most appropriate.

Using equation [2.6-4], you can figure out your savings rate by writing a balance on the transfer of money into and out of your account. The money you deposit from your paycheck is an Input term:

$$\dot{\psi}_{\text{in}} = \left(\frac{\$450}{2\text{ weeks}}\right)\left(\frac{4\text{ weeks}}{\text{month}}\right) = \frac{\$900}{\text{month}}$$

The interest of \$5/month is also an Input term.

All the money you spend is taken out of the system (your account), so these are Output terms:

$$\dot{\psi}_{\text{out}} = \frac{\$75}{\text{month}}(\text{books}) + \frac{\$150}{\text{month}}(\text{food}) + \frac{\$40}{\text{month}}(\text{phone}) + \frac{\$50}{\text{month}}(\text{utilities}) + \frac{\$400}{\text{month}}(\text{rent}) = \frac{\$715}{\text{month}}$$

The net amount of money in the universe (system and surroundings) is constant. No money is generated or consumed; it is just moved between your account and different institutions. You might think that the interest term is a Generation term; however, this is really just the transfer of money from the surroundings (the bank) to the system (your account). The printing of money by the government would be an example of a Generation term, since it would increase the net amount of money in the universe. As a result, money is a conserved property in this (and most) problems.

Substituting these terms into the differential conservation equation gives:

$$\dot{\psi}_{\text{in}} - \dot{\psi}_{\text{out}} = \dot{\psi}_{\text{acc}}^{\text{sys}}$$

$$\frac{\$900}{\text{month}} + \frac{\$5}{\text{month}} - \frac{\$715}{\text{month}} = \dot{\psi}_{\text{acc}}^{\text{sys}}$$

$$\dot{\psi}_{\text{acc}}^{\text{sys}} = \frac{\$190}{\text{month}}$$

Therefore, you are accumulating (saving) \$190 per month in your bank account. ∎

2.6.3 Integral Conservation Equation

Finally, the conservation equation is written in integral form as follows:

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt = \int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}\, dt \qquad [2.6\text{-}5]$$

where

$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt =$ Input between t_0 and t_f, and

$\int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt =$ Output between t_0 and t_f.

The Accumulation between t_0 and t_f can be written as:

$$\int_{t_0}^{t_f}\left(\frac{d\psi}{dt}\right)dt, \int_{\psi_0}^{\psi_f} d\psi, \text{ or } \int_{t_0}^{t_f} \dot{\psi}_{\text{acc}}^{\text{sys}}\, dt$$

For more complete definitions of these variables, refer to the integral accounting equation in Section 2.5.3.

Information on the conditions of the system at either t_0 or t_f or both is usually needed to solve problems using the integral equation. The integral equation [2.6-5] can be reduced to the algebraic equation [2.6-1] for simple systems whose terms do not change with time.

EXAMPLE 2.23 Discharge of a Capacitor

Problem: Capacitors are devices used in biomedical instrumentation to store charge (Figure 2.22). The positive plate of a capacitor has a charge of 10 mC initially ($t = 0$). The plate discharges at a rate i proportional to the net charge on the positive plate (q^{sys}):

$$i = kq^{sys}$$

where k is the proportionality constant 0.5 s^{-1}. To calculate the net charge on the plate at a specified time, should you use an accounting or conservation equation? Is it better to use an algebraic, differential, or integral equation? Consider the capacitor as the system.

Solution: No charge enters the system; however, charge leaves the system. Net charge is neither generated nor consumed in the system, since net charge is a conserved property. Therefore, a conservation equation can be used. Since a rate of movement of charge is specified, the algebraic conservation equation cannot be used. Either the differential or integral conservation equation could be used. Since the problem indicates interest in a specified time, an integral equation is most appropriate. ■

EXAMPLE 2.24 Water in a Bathtub

Problem: Your showerhead sprays water into your bathtub at a rate of 5 kg/min, and water accumulates in the tub at 1.5 kg/min. At what rate does water drain from the tub? If 15 kg of water accumulates in the tub, how long was your shower? After you turn the shower off, how long does it take for the remaining water to drain?

Solution: As shown in Figure 2.23, the system is the tub holding the water, and the extensive property of interest is the quantity of water. The system boundary is drawn to cut across the flow of water into and out of the system. It is assumed that no water enters or leaves the system except as shown (e.g., no water spills over the side of the tub). It is assumed that the drain works at maximum capacity for the entire duration of the shower.

Because the quantity of water is given in flow rates and no time period is specified, a differential equation should be used. The total mass of water is conserved, since water is neither generated nor consumed in a chemical reaction, so the differential conservation equation [2.6-4] can be used.

The rate of the inlet stream of water, $\dot{\psi}_{in}$, is 5 kg/min. Water accumulates in the bathtub at a rate, $\dot{\psi}_{acc}$, of 1.5 kg/min.

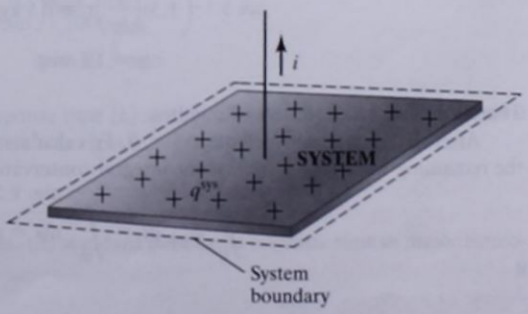


Figure 2.22
Discharging capacitor.

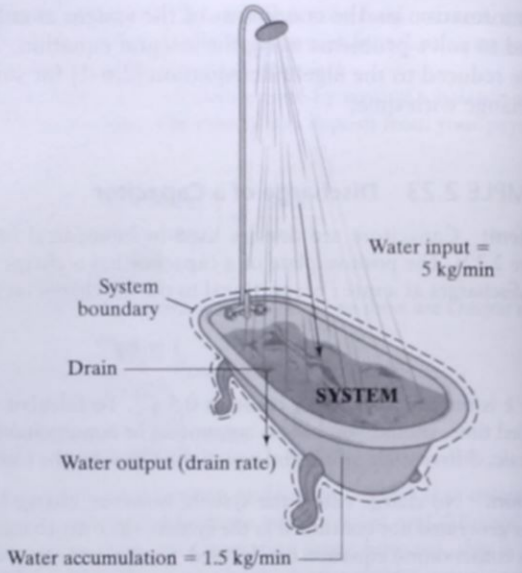


Figure 2.23
Accumulation of water in a bathtub.

$$\dot{\psi}_{in} - \dot{\psi}_{out} = \dot{\psi}_{acc}^{sys}$$

$$5\frac{kg}{min} - \dot{\psi}_{out} = 1.5\frac{kg}{min}$$

$$\dot{\psi}_{out} = 3.5\frac{kg}{min}$$

Thus, the drain rate is 3.5 kg/min.

To calculate the length of the shower, we use the integral mass conservation equation [2.6-5]. The term $\int_{\psi_0}^{\psi_f} d\psi$ is selected to represent the Accumulation term because the accumulated extensive property is known. The difference in the amount of water present in the bathtub between the end and beginning of the shower ($\psi_f - \psi_0$) is 15 kg:

$$\int_{t_0}^{t_f} \dot{\psi}_{in}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{out}\, dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

$$\int_{t_0}^{t_f} \left(5\frac{kg}{min}\right) dt - \int_{t_0}^{t_f} \left(3.5\frac{kg}{min}\right) dt = 15\ kg$$

When $t_0 = 0$ and the equation is integrated with respect to time:

$$\left(1.5\frac{kg}{min}\right) t_f = 15\ kg$$

$$t_f = 10\ min$$

Therefore, the shower lasted 10 minutes.

After the shower is turned off, $\dot{\psi}_{in} = 0$. To calculate the length of time necessary to drain the remaining water, we again use the integral conservation equation, which reduces to:

$$-\int_{t_f}^{t_0} \dot{\psi}_{out}\, dt = \int_{\psi_i}^{\psi_0} d\psi = \psi_f - \psi_0$$

Again, when $t_0 = 0$ and the equation is integrated with respect to time:

$$-\left(3.5\,\frac{\text{kg}}{\text{min}}\right)t_f = -15\ \text{kg}$$

$$t_f = 4.3\ \text{min}$$

Therefore, the accumulated water continues to drain for 4.3 minutes after the showerhead is turned off. ■

EXAMPLE 2.25 Blood Transfusion

Problem: A patient in critical condition is receiving an intravenous blood transfusion to restore blood lost from a gunshot wound. Unfortunately, paramedics were slow to the scene and stopped the bleeding after the patient had already lost 1.6 kg of blood. The attending surgeon estimated that the patient lost blood at a rate of 70 g/min before she or he was stabilized. If a bystander called for help eight minutes after the patient was shot, how long did it take the paramedics to reach the scene of the crime and stop the blood flow?

Solution: The system is the patient's body. The extensive property is mass of blood because the problem tracks blood loss from and replacement to the system. A conservation equation is appropriate since blood is neither created nor destroyed in a chemical reaction. An integral equation is selected because the system involves rates and a fixed time period (even though the time period itself is unknown). To determine the response time, we set up an integral conservation statement:

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\,dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\,dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

The term $\int_{\psi_0}^{\psi_f} d\psi$ is selected to represent the Accumulation term because the amount of lost blood (the extensive property) is known. After the gunshot, the system has an outflow of blood ($\dot{\psi}_{\text{out}}$) and no inflow. The integral conservation equation reduces to:

$$-\int_{t_0}^{t_f} \dot{\psi}_{\text{out}} dt = \psi_f - \psi_0$$

1.6 kg of blood were lost from the system, so $\psi_f - \psi_0$ is −1.6 kg. Inserting the estimated outlet rate of blood, the equation becomes:

$$-\int_{t_0}^{t_f} \left(70\,\frac{\text{g}}{\text{min}}\right) dt = -1.6\ \text{kg}$$

When $t_0 = 0$, the equation is integrated with respect to time:

$$-\left(70\,\frac{\text{g}}{\text{min}}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)t_f = -1.6\ \text{kg}$$

$$t_f = 22.9\ \text{min}$$

We can calculate the paramedic response time (t_r) as the difference between t_f and the time delay to contact the paramedics, which was eight minutes, as follows:

$$t_r = 22.9\ \text{min} - 8\ \text{min} = 14.9\ \text{min}$$

As a comparison, in the United States, the standard paramedic response time in major metropolitan areas is eight minutes.[4] ■

In summary, it is very important to understand that the accounting and conservation equations are parallel across the discussed extensive properties. The same mathematical and computational tools can be applied to the accounting and conservation equations. **The central theme of this book is that two key general equations—accounting and conservation—can be applied to the four main properties of mass, energy, charge, and momentum to solve problems across all fields of bioengineering.**

It is critical to understand the differences between the accounting and conservation equations and to recognize when to apply each. Much of the rest of the book is devoted to setting up conservation and accounting equations for particular systems and to solving the appropriate equations for unknown parameters. **If you are uncertain of which equation to use, write down the accounting equation first.** For particular systems or extensive properties or both, the accounting equation may then reduce to the conservation equation.

The remainder of the book is organized around the four main properties: mass in Chapter 3, energy in Chapter 4, charge in Chapter 5, and momentum in Chapter 6. In Chapter 7, the four properties are integrated in the study of three physiological systems. Mastering the general accounting and conservation equations and learning how to apply them to one extensive property will enable you to transfer your knowledge to other extensive properties.

2.7 System Descriptions

A system or process under investigation may be described using terms that characterize a system. Labeling a system accurately will help you identify the correct governing equation, make appropriate assumptions, and incorporate the correct terms of the accounting and conservation equations.

The terms described below are used widely in engineering and are generally consistent across all fields of engineering. Each term is a quick technical "short hand" for describing an engineering system.

While one of these descriptive terms is often used to describe a system of interest, you need to be careful to also specify the extensive property of interest. Just designating a whole system as open or steady-state may not be true for all the extensive properties, such as mass and energy. (An example of such a system is shown in Example 2.35.) Thus, it is best practice when using these terms to state them in the context of a system and a specific extensive property.

2.7.1 Describing the Input and Output Terms

Recall that Input and Output terms describe the exchange or transfer of extensive properties across the system boundary. These terms also encapsulate all types of movement of extensive properties. When an extensive property crosses the boundary, it is exchanged between the system and the surroundings. There are three terms that describe the characteristics of the Input and Output terms of a system: open, closed, and isolated.

An **open system** is one that exchanges an extensive property with its surroundings through **bulk material transfer**, which is the transfer of an extensive property across a system boundary through quantities of mass. The movement of mass into or out of an open system can transfer mass, energy, charge, or momentum. The extensive property enters or leaves the system by physically crossing the system boundary. In an open system, either the Input or Output terms or both are present (i.e., nonzero). No universally applied reductions to the accounting or conservation equations can be

made. Open systems are very common in bioengineering applications. Open systems are dealt with in Sections 3.4–3.9, 4.5–4.10, 5.5–5.10, and 6.8–6.11.

A **closed system** is one that exchanges an extensive property with its surroundings through means other than bulk material transfer. In a closed system, extensive properties do not cross the system boundary through the transfer of mass. By definition, mass and charge cannot be transferred in a closed system, since they move only by bulk transfer. However, both energy and momentum can be exchanged in a closed system through direct contact or noncontact interactions. Heat is an example of the transfer of energy through direct contact interactions in a closed system. Gravitational force is an example of the transfer of momentum through noncontact interactions in a closed system.

In a closed system describing energy or momentum, no universally applied reductions to the accounting or conservation equations can be made. Closed systems are somewhat common in bioengineering applications. Closed systems are highlighted in Sections 4.4, 5.6, 5.9, and 6.5–6.6.

An **isolated system** is one that does not exchange any extensive properties by any means with its surroundings. No extensive property enters or leaves the system. In an isolated system, both the Input and Output terms are zero. Truly isolated systems are uncommon in biological and medical applications. Isolated systems are explored in Sections 4.4 and 6.7.

The terms open and closed are used to describe the accounting of mass and charge. Since mass and charge can only be transferred through bulk material transfer, the system descriptions of closed, and isolated are synonymous. Engineering convention is to use the term closed (not isolated) to describe systems without transfer of mass or charge. The terms open, closed, and isolated are used to describe the accounting of energy and momentum.

EXAMPLE 2.26 Penicillin Production in a Bioreactor

Problem: Bioreactors are widely used in bioengineering to produce large quantities of vaccines, monoclonal antibodies, antibiotics, pharmaceutical products, and other products. Bioreactors may be operated in a batch, semibatch, or continuous mode.

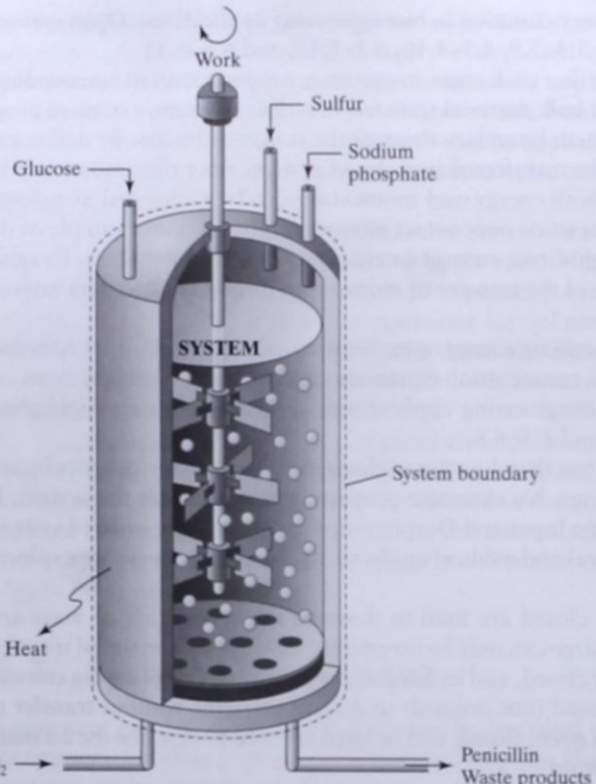
In a **batch** process, the feed materials are added to the bioreactor before the process or reaction begins. No reactants or additional substances are added during operation. Similarly, no products or materials are removed until the process is complete. In a **continuous** bioreactor process, the feed is continuously supplied in input streams, and products and wastes are continuously removed via output streams. In a **semibatch** process, either continuous inlet or outlet streams, but not both, are present.

Figure 2.24 shows a bioreactor designed to produce penicillin. Consider mass and energy as the extensive properties of interest and the bioreactor tank as the system. Materials including glucose, sodium phosphate, sulfur, and oxygen are necessary for penicillin production. Work in the form of stirring is added, and heat is removed from the bioreactor during operation. Identify and describe each bioreactor process as an open, closed, or isolated system.

Solution: The batch process is considered closed with respect to mass as the extensive property because no mass crosses the system boundary (vessel wall) during operation. Reactants are added before the process begins. Penicillin and waste products are removed from the bioreactor after operation is complete.

During the batch process, no energy enters or leaves the system through bulk material transfer, since no mass crosses the system boundary. Since the biochemical reaction produces energy, heat is removed from the bioreactor to maintain a constant operating temperature. Also, work is added when stirring. The batch process should therefore also be considered closed with respect to tracking energy since all energy transfer is through direct contact.

In the continuous and semibatch processes, mass and energy (possessed by the mass) continuously cross the system boundary during operation. Bioreactors that operate in these



eactor.

modes are open systems with respect to mass and energy, since material transfers both mass and energy. These two modes of operation are the most common in bioprocessing.

A bioreactor configuration that is isolated with respect to energy is uncommon. Because controlling the temperature in the bioreactor is critical to successful operation, heat is often added or removed. In addition, work as stirring is usually necessary to keep the system well mixed. ∎

While it is well known that Sir Alexander Fleming was the first scientist to recognize the importance of a specific strain of mold in 1928, it is not so well known that the team of Florey, Chain, and Moyer engineered the revolutionary drug penicillin and forever altered how antibiotics were used and produced to treat bacterial infections.

In 1939, Florey and Chain were the first to show that mice infected with multiple strains of bacteria could be successfully cured with penicillin, implicating its potential in treating disease. Their team soon developed a powder form of penicillin—the first antibiotic. In 1941, Moyer developed a method to increase yields of the mold, which was incredibly difficult to purify. Moyer's method allowed massive quantities of the drug to be produced and later led to large-scale production.

Penicillin's success in treating millions of wounded American soldiers during World War II paved the way for antibiotics research, as well as the industrialization of U.S. pharmaceutical companies. As methods of purifying the drug improved, the price per dose dropped rapidly, from $20 in 1943 to 55 cents in

Figure 2.25
Astronauts experiencing a force caused by Earth's gravitational field. The force her body exerts on the ground ($\vec{W}$) is equal in magnitude to and opposite in direction from the normal force the ground exerts on her body ($\vec{n}$). Photo courtesy of NASA.

EXAMPLE 2.27 Momentum on Earth and in Space

Problem: An astronaut is standing completely still on the Earth's surface while she waits to board a spacecraft on her mission to Mars. Except for Earth's gravitational field, no other external forces act on her body (Figure 2.25). Assume that during the mission to Mars, the effect of gravitational fields from celestial bodies is negligible, and that her body does not feel the effects of any other contact or noncontact forces (Figure 2.26).

Consider her body as the system and momentum as the extensive property of interest. Is the system open, closed, or isolated on Earth? In space? Neglect unmentioned sources of momentum transfer (e.g., electrical and magnetic fields) to her body.

Solution: On Earth's surface, no momentum carried through bulk material transfer crosses the system boundary (the surface of her body). However, Earth's gravitational field exerts a noncontact force on the astronaut's body. Through this force, the system exchanges momentum with the surroundings. Thus, the system is closed but not isolated.

For a similar analysis in space, no bulk material transfer into or out of the system occurs. We assume that between Earth and Mars, the astronaut is sufficiently far from each planet that any gravitational force acting on her body is negligible. Furthermore, in the absence of gravity, the astronaut floats around freely within the spacecraft, so no other contact forces act on her body. When neglecting other sources of momentum transfer, her body may be considered an isolated system with respect to momentum, since no momentum of any kind is transferred to her body. ■

EXAMPLE 2.28 Heating Water to a Boiling Temperature

Problem: A tea kettle holding 600 g of room temperature water is set on a hot stove (Figure 2.26). The stove transfers energy to the kettle at a rate of 640 J/s until the water reaches 100°C, at which point it can boil. How long will it take for the kettle to reach 100°C, immediately before it begins to boil? Consider the kettle as a system and energy as the extensive property. Determine if the kettle is an open, closed, or isolated system with respect to energy.

Note that the enthalpy change of vaporization would be involved if we were to consider that some of the water would probably reach the boiling point and begin to change phase before the entire kettle of water has reached 100°C. While not entirely realistic, here it can be assumed that the water collectively warms to the boiling point before any of it begins to boil.

Solution: There is no bulk transfer of material to the kettle since no mass crosses the system boundary. However, energy enters the system by means of direct contact. Specifically, energy

n a kettle of

is transferred from the stove to the kettle as heat. As a result, the kettle is considered a closed system with respect to energy.

The first step is to calculate the amount of energy necessary to raise 600 g of water from room temperature (22.5°C) to 100°C. Using the heat capacity of water, the amount of energy used to raise the temperature as specified is determined as:

$$600\ \text{g}\left(4.184\ \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 22.5^\circ\text{C}) = 194{,}600\ \text{J}$$

Since total energy can be neither created nor destroyed, a conservation equation is used. An integral conservation equation is selected because we are solving for a particular time period. The term $\int_{\psi_0}^{\psi_f} d\psi$ is selected to represent the accumulation term because the desired total accumulation of extensive property is known.

Energy enters the kettle at a rate of 640 J/s. There is no energy output from the system (i.e., the kettle) that is specified. 194,600 J of energy must accumulate in the water in the system. The integral conservation equation is solved as:

$$\int_{t_0}^{t_f} \dot{\psi}_{\text{in}}\, dt - \int_{t_0}^{t_f} \dot{\psi}_{\text{out}}\, dt = \int_{\psi_0}^{\psi_f} d\psi = \psi_f - \psi_0$$

$$\int_{t_0}^{t_f} \left(640\frac{\text{J}}{\text{s}}\right) dt = 194{,}600\ \text{J}$$

When $t_0 = 0$, the equation can be integrated with respect to time:

$$\left(640\frac{\text{J}}{\text{s}}\right) t_f = 194{,}600\ \text{J}$$

$$t_f = 304\ \text{seconds} = 5.1\ \text{min}$$

Thus, 304 seconds or about 5.1 min is needed to warm 600 g of water from room temperature to 100°C in a tea kettle. ∎

2.7.2 Describing the Generation and Consumption Terms

Recall that Generation and Consumption terms describe the creation and destruction of an extensive property within a system. These terms are present in accounting equations describing nonconserved properties, such as moles of a chemical species,

negative charge, or mechanical energy. When an extensive property is generated or consumed in a system, the surroundings do not lose or gain, respectively, the equivalent amount of that property. Instead, the extensive property is created or destroyed in both the system and the universe. This is the main discriminator for characterizing a term as an Input or Output term or as a Generation or Consumption term. When a Generation or Consumption term is present, the extensive property is generated or consumed in the system and the surroundings. When an Input or Output term is present, the extensive property is transferred between the system and the surroundings. When a Generation or Consumption term is present, an accounting equation must be used. Two main processes constitute generation or consumption of an extensive property. The first is a chemical reaction, and the second is an energy interconversion.

When a chemical species reacts to produce a new product, a quantity of the species' total moles or mass is destroyed in the system and the universe. Chemical reactions are considered in the accounting of mass, charge, and energy.

Chemical reactions are defined broadly in this book to include both the rearrangement of molecules between compounds and species dissociation or electrochemical reactions, as well as the transfer of electrons and other atomic particles in nuclear reactions.

A **reacting system** is one in which at least one biochemical or chemical reaction is taking place. When a system is reacting, accounting equations are required to deal with nonconserved extensive properties, such as individual species moles. Conservation equations are appropriate only when applied to conserved properties, such as total mass. Reacting systems requiring the accounting equation are discussed in Sections 3.8–3.9 and 5.9–5.10. In Sections 4.8–4.9, systems with chemical reactions are presented; however, as total energy is conserved, conservation equations are used.

In a **nonreacting system**, no biochemical, chemical, or other reactions are taking place. The Generation and Consumption terms in the accounting equation can be set to zero if a system is both nonreacting and without energy interconversions. Nonreacting systems are discussed in Sections 3.4–3.7, 3.9, 4.4–4.7, 4.10, 5.5–5.8, and 6.5–6.11.

The second process constituting the generation or consumption of an extensive property is the interconversion of different types of energy (e.g., mechanical, thermal, and electrical). In a system with **energy interconversion**, one form of energy is converted to another form, so the first form of energy has depleted in the system and in the universe. For example, when mechanical energy is converted to heat, such as when frictional loss occurs, the total amount of mechanical energy in both the system and the universe has decreased. Therefore, these types of energy interconversions are considered in the Generation and Consumption terms. When tracking for a specific type of energy (but not total energy), the accounting equation must be used. Systems with energy interconversion requiring the accounting equation are discussed in Sections 5.6, 5.8, 5.10, and 6.11–6.13.

Table 2.4 summarizes the types and classification of Input/Output and Generation/Consumption terms covered in this textbook. Notice that all extensive properties without any Generation or Consumption terms are conserved properties. That is, the extensive properties of total mass, elemental mass, elemental moles, total energy, net charge, linear momentum, and angular momentum can be neither created nor destroyed in the system and in the universe. **It is appropriate to use the conservation equation for conserved properties. For all other extensive properties, the accounting equation, which contains the Generation and Consumption terms, must be used to describe the system.**

TABLE 2.4

Summary of Presence of Terms in Accounting Equation

Accumulation	Input − Output		+ Generation − Consumption	
Extensive property	Bulk material transfer	Direct and non-direct contacts	Chemical reactions	Energy interconversion
Total mass	X			
Species mass	X		X	
Elemental mass	X			
Total moles	X		X	
Species moles	X		X	
Elemental moles	X			
Total energy	X	X		
Thermal energy	X	X		X
Mechanical energy	X	X		X
Electrical energy	X	X		X
Net charge	X			
Positive charge	X		X	
Negative charge	X		X	
Linear momentum	X	X		
Angular momentum	X	X		

EXAMPLE 2.29 Penicillin Production in a Bioreactor II

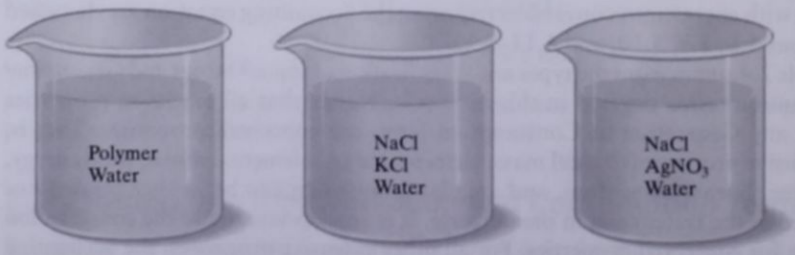
Problem: As discussed in Example 2.26, bioreactors are used for the production of a wide variety of biological and pharmaceutical products. A multistep process isolates the product after it leaves the bioreactor by using physical separation methods to remove waste. Identify the processes in both the bioreactor and separation systems as reacting or nonreacting.

Solution: In the bioreactor, different chemical constituents of the feed, such as glucose and oxygen, undergo biochemical reactions and are converted to penicillin and waste products. Thus, the bioreactor is a reacting system.

Methods of product isolation are numerous, including liquid–liquid extraction, vacuum distillation, and precipitation. Product isolation usually involves the physical separation of substances, not any chemical reactions between components; thus, the separation system is nonreacting. ∎

EXAMPLE 2.30 Solutions in a Beaker

Problem: A chemistry student has three beakers (Figure 2.28). In the first, she or he adds a block of inert polymer to water. In the second, she or he mixes two salts, NaCl and KCl, in



ing chemical
f which react.

water. In the third, she or he mixes NaCl and $AgNO_3$ in water. Consider the charge in each beaker as the extensive property of interest. Identify each beaker as a reacting or nonreacting system.

Solution: In the first beaker, the polymer does not undergo any type of chemical or dissociation reaction. Therefore, this system is nonreacting with respect to charge.

In the second beaker, the two salts dissolve in water and their molecules dissociate to form Na^+, Cl^-, and K^+ ions. These ions mix and intersperse throughout the solution. Because the salts dissociate into charged species, this system is considered reacting with respect to charge.

In the third beaker, a double replacement reaction occurs in which the cations exchange anionic partners:

$$NaCl(aq) + AgNO_3(aq) \longrightarrow NaNO_3(aq) + AgCl(s)$$

These specific compounds chemically react to form a silver chloride precipitate, so the third beaker is also a reacting system with respect to charge. ■

EXAMPLE 2.31 Cardiac Mechanical Energy Interconversion

Problem: Consider the left atrium and left ventricle of the heart. In between contractions, oxygenated blood flows into the left ventricle through the left atrium. During a contraction, this oxygenated blood is pumped from the left ventricle into the aorta. As blood flows through the left ventricle, mechanical energy passes through as well. Additionally, the force of the left ventricle's contraction imparts additional mechanical energy to the blood as it leaves the chamber. Consider the momentum of the blood in the left ventricle. Identify which terms are present in a mechanical energy accounting equation.

Solution: The extensive property is mechanical energy. The system is the left ventricle. The system boundary is the edge of the ventricle. Note that all terms are rate of mechanical energy.

As blood enters and leaves the ventricle through bulk material transfer, it carries mechanical energy, so both Input and Output terms are present in the accounting equation. Energy interconversion, captured in the Generation and Consumption terms, happens in two ways. First, there is the contractile force of the heart on the blood that is captured as work. Second, mechanical energy is lost from the system due to friction. ■

2.7.3 Describing the Accumulation Term

The Accumulation term describes the net gain or loss of an extensive property contained within a system. When an Accumulation term is present, the amount of extensive property in the system has changed during the time period of interest. There are two terms that describe the characteristics of the Accumulation term of a system: steady-state or dynamic.

Steady-state is a condition in which the values of all the variables in a system (e.g., temperature, pressure, volume, and flow rate) do not change with time, although minor fluctuations about constant mean values may occur. Let us illustrate this by using a photography analogy. If you take multiple, imaginary "snapshots" of a steady-state system over a time period, each snapshot should look the same as the previous one. The initial and final conditions and all the intermediate snapshots of the system are identical or nearly so. The snapshots show that no quantity of the extensive property has collected in the system.

A common misconception is that systems with Input and Output terms cannot be in steady-state. This is false. Systems can have Input and Output terms, as well as

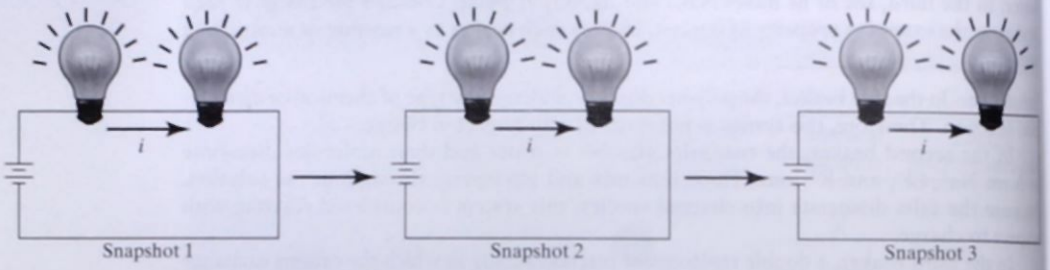


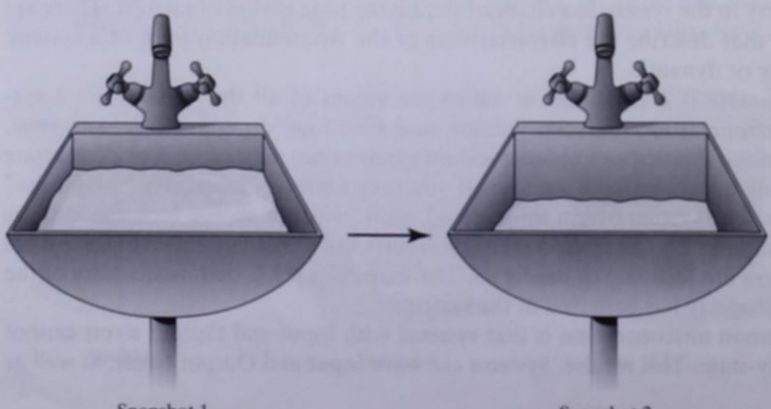
Figure 2.29
Circuit in steady-state.

Generation and Consumption terms, and still operate in steady-state. What makes a system steady-state is that it does not change over time, nor is there positive or negative accumulation of the extensive property within the system of interest.

For example, if an extensive property is continuously flowing into the system at the same rate it flows out, such as when water flows at a constant rate through a pipe, the amount of extensive property in the system remains the same. Thus, snapshots of the system (i.e., pipe) look the same, and the system is in steady-state. Consider the electrical energy in a circuit with two light bulbs. Current flows through the circuit at a constant rate and there is no accumulation in the system (Figure 2.29). As a result, every "snapshot" of the system over time looks the same and the system is in steady-state. In a steady-state system, the Accumulation term is zero. Steady-state systems are highlighted in Sections 3.4–3.8, 4.5–4.9, 5.5–5.6, 5.10, 6.5–6.8, and 6.11.

Dynamic state, also called **unsteady-state** or **transient**, is a condition in which the values of at least one variable in a system change with time. Using the photography analogy, if you take multiple, imaginary snapshots of an unsteady-state system, the snapshots should look different from one another. In a dynamic system, there is a positive accumulation (gain) or a negative accumulation (loss) of an extensive property. In other words, there is more (or less) of the extensive property of interest when considering the initial and final times.

Consider a sink of water that is draining (Figure 2.30). With water as the extensive property, the system is dynamic. As another example, there is positive accumulation of energy in a pot of water when it is warmed on the stove. In both of these



.30
g sink is a dynamic

examples, snapshots of the system over time look different. Snapshots of the sink show less water over time, while snapshots of the pot show increasing energy contained in the water.

Because the initial and final conditions of the system are not equivalent, the accumulation term is nonzero. Thus, a positive or a negative accumulation of an extensive property means that no universally applied reductions to the accounting or conservation equation can be made. Dynamic systems are highlighted in Sections 3.9, 4.4, 4.10, 5.7–5.9, and 6.9.

Whether a system is steady-state or dynamic depends greatly on the time scale over which you are examining it. Consider the heartbeat of a person during his teenage years. If you looked at snapshots of the electrical activity of his heartbeat on an electrocardiograph (Figure 2.31), where each wave corresponds to the electrical depolarization or repolarization of a compartment of the heart, the snapshots would look virtually identical over the course of a year. Thus, the system (the heart) is in a steady-state condition during the course of one year. However, if you took several snapshots within a single heartbeat, each snapshot (i.e., mV value) would look different from the previous one; the heart is in a dynamic state during the timeframe of one heartbeat.

EXAMPLE 2.32 Solutions in a Beaker II

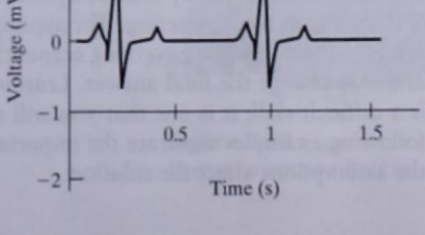
Problem: Consider the mixing of two different salts, NaCl and KCl, in a beaker of water (Example 2.30). Consider the time period over which the chemistry student adds and mixes the salts. If charge is the extensive property of interest in the beaker system, is the system steady-state or dynamic?

Solution: Initially, the beaker contains no charged species, only water, NaCl, and KCl. After dissociation (final condition), the beaker contains water and Na^+, Cl^-, and K^+ ions. Using the photography analogy, comparing the snapshots over the time period of interest reveals that the charge contents in the beaker change with time. The system contains more charged ions at the end than at the beginning, indicating a positive accumulation. Thus, the system is dynamic. ■

EXAMPLE 2.33 Freshman Fifteen

Problem: Students matriculating into college are often challenged by the "freshman fifteen," when they gain an average of 15 lb_m by the end of their freshman year. The mass gain is often attributed to the lack of exercise and the late-night pizzas, sodas, and snacks needed to cram for exams.

Josh's mass is 175 lb_m when he matriculates. By the end of his freshman year, he has gained 15 lb_m. During the summer, Josh works out every day and makes a conscious effort to eat healthy foods. By the beginning of his sophomore year, he is back to 175 lb_m, and he is able to maintain that mass until his graduation.



Consider Josh as the system and his mass as the extensive property of interest. Consider his freshman year. Is Josh a steady-state or dynamic system? What about for the time period of a single day during his freshman year? Is he a steady-state or dynamic system over his collegiate career?

Solution: This is one example of how changing the time period can change your assumptions about how the system is described.

At matriculation (initial condition), Josh's mass is 175 lb_m. At the end of his freshman year (final condition), Josh's mass is 190 lb_m. Because the initial and final conditions of the system are different and mass has collected in the system, Accumulation is nonzero and the system is dynamic over Josh's first year of college.

For a single day in his freshman year, Josh's mass may fluctuate slightly as he eats, drinks, and excretes over a 24-hour period. However, using the photography analogy, snapshots of Josh during this time period would not change drastically and would look virtually identical. His overall mass is not expected to change much; that is, the change in mass in one day is negligible in comparison to his overall mass. Thus, Josh is considered a steady-state system over a 24-hour period.

Over his collegiate career, Josh's mass mostly remains the same due to his attention to diet and his regular exercise routine. At his matriculation (initial condition) and at his graduation (final condition), he is 175 lb_m. Although he experienced mass accumulation during his freshman year and mass loss the following summer, taking multiple snapshots of Josh over the time period of interest would reveal snapshots that mostly looked the same. Thus, Josh's "freshman fifteen" can be considered a fluctuation about his mean mass value. Josh is considered a steady-state system over his collegiate career. ∎

EXAMPLE 2.34 University Enrollment

Problem: A small university enrolls 3200 undergraduate students. During the late spring of every year, 800 senior students graduate from the university with their bachelor's degree. Each fall, 600 students matriculate as freshmen and 200 undergraduates from other universities transfer to the school. Is the university a steady-state or dynamic system with respect to the number of undergraduate students?

Solution: The extensive property is the number of enrolled students. The system is the university and the registrar keeps track of the enrolled students. During the academic year, enrollment is constant at 3200 students; thus the system is at steady-state. During the summer, there are 2400 students since 800 graduated. During the summer months, the system is also at steady-state.

Looking at a small time period, for example, July to October, the number of enrolled students is dynamic because there is an accumulation of 800 matriculants on campus in the early fall. From a practical perspective, a university with enrollment of 3200 from year to year would be considered at steady-state. Note that this steady-state example has Input terms (matriculating freshmen and transfer students) and Output terms (graduating seniors). ∎

2.7.4 Changing Your Assumptions Changes How a System Is Described

When analyzing complex systems, it is essential to examine the three categorical distinctions discussed in the previous sections. Learning to assess systems (e.g., open, closed, or isolated; steady-state or dynamic) is crucial to accurate and thorough problem solving in bioengineering. The assumptions you make about a system may profoundly affect how the governing accounting and conservation equations are applied and may change the final answer. Learning to make good engineering assumptions is a difficult skill; it is one that you will master as you mature as an engineer. The following examples illustrate the importance of how the system is set up and how the assumptions affect the solution.

EXAMPLE 2.35 Training for a Cycling Event

Problem: Robbie decides to enter a cycling event to help raise money for a charity that aids multiple sclerosis patients and funds research for treatment and cures. In the summer, he trains for 45 minutes each day and stops to drink 200 mL of water every 15 minutes. Consider Robbie's body as the system. Consider the extensive properties of mass and energy. Is the system open or closed? Is he at steady-state? During warm-up, how would you best describe Robbie as a system?

Solution: *Mass* (Figure 2.32): When Robbie drinks water, mass enters the system. Robbie loses mass (e.g., water and salts) through perspiration. Because mass enters and leaves the system, the system is open. Since Robbie only drinks every 15 minutes during the 45-minute training ride, the Input term is not continuous with time; therefore, the accumulation of water in Robbie's system on a short time scale (e.g., 5 min) is not constant with time. Because Robbie's mass at the initial and final conditions of the ride is different, he is in transient mode. The same analysis holds true during Robbie's warm-up. If Robbie drank one sip of water every few seconds and his physiological rates (e.g., heart, blood pressure, and perspiration) were constant for a period of time, you might choose to assume that he is at steady-state.

Energy (Figure 2.33): Stored energy in Robbie's body is converted to work to generate power for him to cycle. Assuming that the water he drinks has no caloric value, energy does not enter his system from food or other sources during cycling, but Robbie does lose a tremendous amount of heat during exercise. The work and heat we have just described, which are forms of energy, leave the system, but not through bulk material transfer. From this perspective, the system describing energy is closed, but not isolated. However, when Robbie perspires, energy is lost in the bulk material transfer of water through the skin's surface. Additionally, Robbie loses energy when he breathes. With these considerations, the system should be classified as open. If Robbie's vital signs (e.g., body temperature) and his rate of perspiration do not change with time, you might think that he is at steady-state. However, the initial and final conditions of Robbie's energy status are definitely different, since his energy reserves have been depleted. Therefore, the best assumption is that Robbie is in transient, or dynamic, mode. During warm-up, his physiological characteristics change with time; therefore, he is definitely in transient mode.

The description of a system depends on the extensive property of interest (e.g., mass or energy) as well as the time scale. In Chapters 3 and 4, quantitative examples of mass and energy transfer in the human body are presented. Also, you will learn how cells metabolize food to generate heat, work, and stored energy. ■

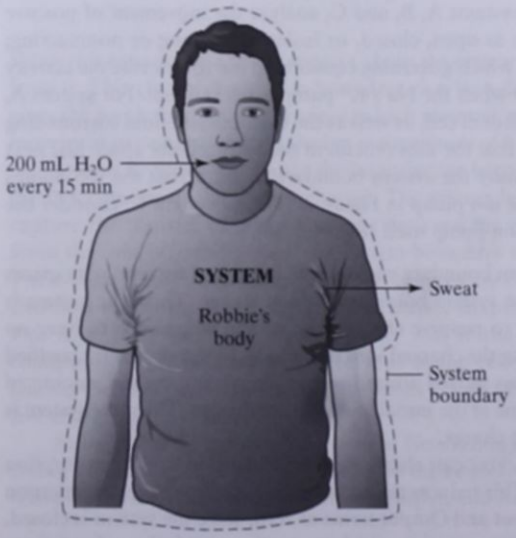


Figure 2.32
Mass balance on Robbie.

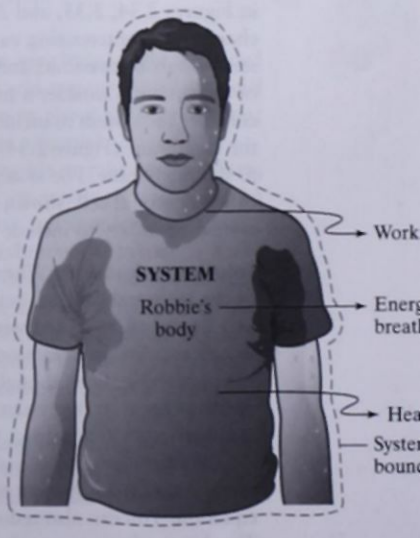


Figure 2.33
Energy balance on Robbie.

Recall that a system is defined by its boundary. Defining the system of interest allows you to make certain assumptions before you begin to solve a problem. Thus, changing a system by moving its boundary can change your assumptions, such as whether a system is steady-state or dynamic.

How you define the system characterizes the system itself. This in turn affects which governing equations are appropriate to use, as well as what reductions to these equations are suitable to the system and the problem. In most cases, the system should be defined such that the movement of the extensive property being examined can be tracked across the system boundary. Often, **the system boundary should be drawn to cut across any inputs and outputs to the system.** The importance of placement of the system boundary relative to movement of extensive property is highlighted in the following two examples.

EXAMPLE 2.36 Action Potential in Neurons

Problem: Across the plasma membrane of most neuronal cells, there is a charge gradient. This gradient is established because the volume inside the cell membrane has a negative electrical charge relative to the extracellular space adjacent to the membrane. The gradient keeps the cell at its resting electrical potential, approximately −90 mV in neurons, which is necessary for signal transduction. To maintain this membrane potential, ion pumps and channels facilitate the movement of charge into and out of the cell. For example, the sodium/potassium pump pushes two potassium ions into the cell for every three sodium ions it pumps out, resulting in a net loss of positive charges from the intracellular space. Potassium channels in the membrane allow K^+ ions to diffuse back out of the cell in response to the excess of K^+ ions inside the cell.

Certain external stimuli, such as electrical stimuli (e.g., sensory events, such as touch or temperature sensation), mechanical stimuli (e.g., stretching), and neurotransmitters (e.g., acetylcholine), cause the rapid influx of sodium and potassium ions into the cell. If the membrane potential reaches a threshold voltage of about −65 mV, an action potential is generated and the membrane completely depolarizes in a period of less than 1 ms. When the membrane depolarizes, it becomes highly permeable to sodium ions, which rapidly move into the cell to neutralize the charge inside, collapsing the charge gradient across the membrane. This ability to rapidly alter the membrane potential gives neurons the ability to transmit signals across the body.

Consider the three system boundaries for the Na^+/K^+ pump in the neuron as shown in Figures 2.34, 2.35, and 2.36. For systems A, B, and C, analyze the movement of positive charge by characterizing each system as open, closed, or isolated; reacting or nonreacting; steady-state or dynamic; and deciding which governing equation to use to describe the activity of the system. Consider a time period when the Na^+/K^+ pump is operational. For system A, consider the system to include one neuronal cell, as well as the extracellular fluid surrounding the membrane (Figure 2.34). Assume that the ions remain in the extracellular space just next to the membrane. For system B, consider the system boundary to cut across the ion pumps of the neuronal cell (shown as a single ion pump in Figure 2.35). For system C, consider the system boundary to include only the ion pump itself (Figure 2.36).

Solution: *System A:* When the system boundary encloses the intra- and extracellular spaces of the neuronal cell, no ions cross the system boundary by any means. Thus, the system is best described as closed with respect to positive charge. The system is assumed to have no significant chemical reactions involving the charged ions of interest, so the system is classified as nonreacting. Even though charge has moved around within the system, the net amount of positive charge in the system is the same at the initial and final conditions. Thus, the system is at steady-state with respect to positive charge.

Since the system is nonreacting, you can eliminate the Generation and Consumption terms from the accounting equation. This reduces the accounting equation to the conservation equation. You can also reduce the Input and Output terms to zero, since the system is closed. The Accumulation term of positive charge is zero, which is consistent with the reductions to the governing equation:

$$\psi_{acc}^{sys} = 0$$

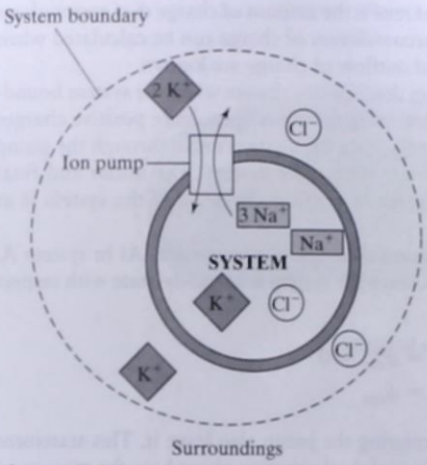


Figure 2.34
System that includes cell and immediate extracellular space.

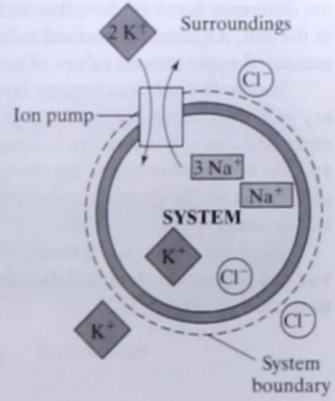


Figure 2.35
System containing only the cell.

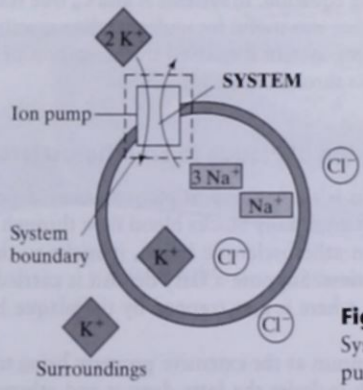


Figure 2.36
System containing one ion pump.

While this equation is a true statement about the system of interest, very little insight is gleaned from it. This choice of a system boundary that includes the cell and regions outside the cell gives the result that nothing changes, which counters what we know about the very dynamic movement of positive charges during maintenance of a membrane potential.

System B: When we change the boundary to become the tangible membrane boundary, the descriptions of the system change. The system boundary cuts across the ion pump to capture the transfer of positive charge between the intracellular and extracellular spaces. Since the ions physically cross the system boundary of the membrane, the system is open. The system is still nonreacting, since no charges are created or destroyed within it through chemical reactions. When the pump is active, snapshots of the cell show the loss of Na^+ ions from the system and the gain of K^+ ions in the system. Because the initial and final conditions of the positively charged ions in the cell are different, the system is dynamic. During the time period of interest, there has been a negative accumulation (or loss) of positive charge in the system.

As in system A, you can eliminate the Generation and Consumption terms from the governing accounting equation, reducing it to the conservation equation. However, because the system is open and dynamic, the Input, Output, and Accumulation terms are nonzero:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys}$$

Thus, the accumulation of positive charge in the system is equal to the difference of the Input and the Output. This choice of system boundary gives us some useful information, namely that

the difference between the influx and outflow of ions is the amount of charge that accumulates in the cell. An actual numerical value for the accumulation of charge can be calculated when measured experimental values of net influx and outflow of charge are known.

System C: Last, we examine how the system descriptions change when the system boundary includes only an ion pump. Again, the nonreacting system is open, since positive charges enter and leave, crossing the ion-pump boundary. Since the charges move through the pump and do not remain there, no charges accumulate within the system. The initial and final conditions of the pump are the same with respect to positive charge, and the system is at steady-state.

As in the other two systems, the conservation equation is appropriate. As in system A, you can also set the Accumulation term to zero, since the system is at steady-state with respect to positive charge:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys} = 0$$
$$\psi_{in} = \psi_{out}$$

This equation states that all positive charges entering the pump also leave it. This statement certainly makes sense, although it provides little useful information about how the movement of positive ions affects the membrane potential of the cell.

To reiterate, where you define the system boundary makes a significant impact on the reductions to the accounting equation. In systems A and C, true statements describing the systems were derived, but neither was useful for understanding membrane potential. In contrast, the system boundary defining system B enabled the derivation of an equation that captured the behavior of positive ions through a Na^+/K^+ pump. ∎

EXAMPLE 2.37 Collision of Plaque in an Atherosclerotic Vessel

Problem: Atherosclerosis is the buildup of plaque—fatty deposits, cholesterol, calcium, and other substances—that eventually blocks blood flow through the arteries. Usually when a material collides with an atherosclerotic lesion, it sticks to the plaque buildup already present and hardens over time. Suppose a fatty deposit is carried by the blood at a known velocity to the lesion site, where it gets trapped by the plaque buildup. Ignore the effects of gravity.

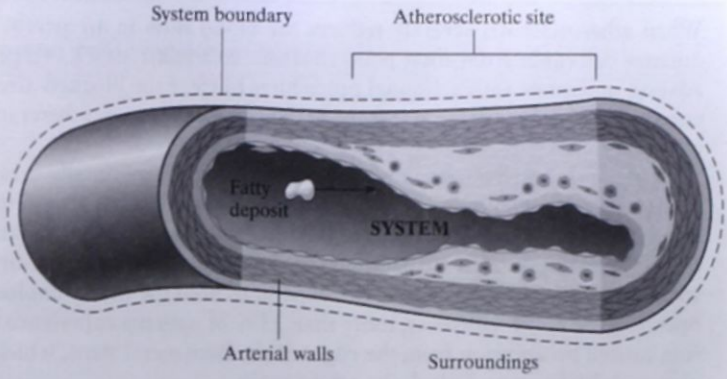
Consider linear momentum as the extensive property being tracked. Consider the entire artery (Figure 2.37), which includes the fatty deposit and atherosclerotic site. How would you characterize this system? What governing equation is appropriate to use to find the linear momentum after the collision? What reductions to the governing equation can you apply? What happens if you change the system boundary so that the system is only the atherosclerotic lesion site (Figure 2.38)?

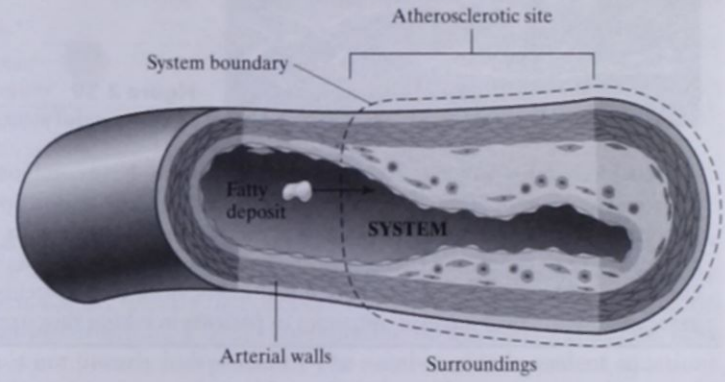
Solution: When the system boundary encloses the entire artery as the system, momentum is not transferred to the system by the movement of mass across the system boundary. Because the extensive property of interest does not enter or leave through bulk material transfer, the system can be characterized as closed. Also, no external forces, such as gravity, act on the system. In the absence of any momentum transfer mechanism, the system is considered isolated.

By definition, linear momentum is a conserved property and cannot be generated or consumed through a chemical reaction or through energy interconversion. So, even though chemical reactions may follow after a fatty deposit collides with the plaque, any such reaction will not contribute to the Generation or Consumption term in a linear momentum accounting equation.

Because the system is isolated, both the Input and Output terms are eliminated. The momentum of the system does not change between the initial and final conditions, making the system steady-state with respect to momentum. Thus, the Accumulation term is equal to zero:

$$\psi_{acc}^{sys} = \psi_f - \psi_0 = 0$$
$$\psi_0 = \psi_f$$





Thus, the final momentum is equal to the initial momentum of the system.

When we change the system boundary so the system includes only the atherosclerotic region, the descriptions of the system change. The system is now open instead of isolated, since the fatty deposit crossing the boundary carries momentum into the system through bulk material transfer. The momentum of the system changes during the time period. Since the initial and final conditions are not equal, the system is dynamic.

As for the system defined as the atherosclerotic region, we use the conservation law as the governing equation since linear momentum is a conserved property. Since this system is open and dynamic, the Input, Output, and Accumulation terms should all be written in the initial formulation of the equation. Given that no mass, and hence no momentum, leaves the system boundary, the Output term is zero. The initial momentum, ψ_0, of the system is zero, since it has no initial velocity:

$$\psi_{in} - \psi_{out} = \psi_{acc}^{sys} = \psi_f - \psi_0 = 0$$

$$\psi_{in} = \psi_f$$

Thus, the momentum of the system at the final condition (ψ_f) is equal to the momentum added to the system (ψ_{in}).

If you were to assign masses to the fatty deposit and the plaque buildup and a velocity to the fatty deposit, the governing equations would reduce similarly. In this case, the two proposed system boundaries yield similar information, even though the system boundaries were established at different places. ■

When atherosclerosis severely reduces the blood flow in an artery, the consequences can range from chest pains to stroke to a heart attack. To prevent such adverse outcomes, interventional procedures to clear the blocked artery are necessary. These procedures range from oral medicines to transcatheter intervention to surgery.

One of the most common treatment procedures is minimally invasive angioplasty. A deflated balloon-tip on a catheter is inserted into a vein in the patient's groin, guided through the vessels to the blockage, and inflated to flatten out the plaque against the artery wall. Often surgeons also use the balloon-tip to insert a stent, a wire-mesh skeleton that can expand to hold the newly unblocked artery open (Figure 2.39). However, more than 25% of patients experience tissue scarring caused by abrasion from the edges of the bare metal stent, which can cause plaque to build up and block the artery again.

Figure 2.39
Fully expanded stent.

To combat this problem, bioengineers and surgeons have combined efforts to improve the stent. In 2003, the U.S. Food and Drug Administration approved one potential solution: a drug-eluting stent that reduces scar tissue growth. While they are relatively new and their long-term performance has not been as thoroughly evaluated as the performance of bare metal stents, drug-eluting stents have been associated with fewer adverse outcomes in patients in whom they are placed.[3]

2.8 Summary of Use of Accounting and Conservation Equations

As discussed in Section 2.4, some extensive properties should be counted using accounting equations, whereas others can be counted using conservation equations. Table 2.5 summarizes which equations can be used to count specific extensive properties. Note that the accounting equation is *always* valid, while the conservation equation is appropriate only for those extensive properties that are conserved. Because the accounting equation is always valid, we use it as the starting point for a majority of the example problems in this book.

Determining when and how to employ conservation and accounting equations in a variety of medical and biological systems is the major focus of Chapters 3–7. Frequently, solving a problem is not what makes bioengineering difficult; rather, it is setting up the system, evaluating the system parameters, and writing the appropriate accounting and conservation equations that is challenging. Regardless of your mathematical and problem-solving skills, an inappropriately drawn system or an invalid equation will lead to incorrect answers. Thus, making you comfortable with defining a system and writing its corresponding accounting or conservation equation is a major goal of this book.

The number of **scalar equations** is the number of equations that can be written for an extensive property in a system. For example, with elemental mass, one equation can be written for each element n in the system. Linear and angular

TABLE 2.5

Use of Accounting and Conservation Equations

Name of property	Accounting equation universally valid?	Conservation equation universally valid?	Number of scalar equations
Total mass	Yes	Yes	1
Species mass	Yes	No	m, for m species
Elemental mass	Yes	Yes	n, for n elements
Total moles	Yes	No	1
Species moles	Yes	No	m, for m species
Elemental moles	Yes	Yes	n, for n elements
Total energy	Yes	Yes	1
Thermal energy	Yes	No	1
Mechanical energy	Yes	No	1
Electrical energy	Yes	No	1
Net charge	Yes	Yes	1
Positive charge	Yes	No	1
Negative charge	Yes	No	1
Linear momentum	Yes	Yes	3
Angular momentum	Yes	Yes	3

Adapted from Glover C, Lunsford KM, and Fleming JA, *Conservation Principles and the Structure of Engineering*, 4th ed. New York: McGraw-Hill, Inc., 1994.

momentum each have three scalar equations, since there are three directional axes (e.g., x, y, z in Cartesian; r, θ, ϕ in spherical). **Independent equations** are those that are mathematically linearly independent from one another. (If one equation in a group can be formed by a linear combination of the other equations, then that equation is not linearly independent.) The number of independent equations for a system is equal to the number of linearly independent equations that can be written for a system.

Most biological systems contain many different components or chemical constituents. Many different types of mass and mole accounting equations can be written to describe a system. One species mass accounting equation can be written for each of m species in a multicomponent situation. One conservation equation for total mass can also be written. Thus, for a system with m species, $m + 1$ mass balance equations can be written. Of these, only m equations are linearly independent. One elemental mole accounting equation can be written for each of n elements in a system. One total mole accounting equation can also be written. Of these $n + 1$ mole balance equations, only n are linearly independent. The situation is similar when considering all forms of species and elemental mass and mole balance equations.

One scalar accounting or conservation equation can be written for the properties of total energy, thermal energy, mechanical energy, and electrical energy. Although they are linearly independent from one another, these equations are typically not used together to solve a problem.

One scalar conservation equation can be written for net electric charge, and one accounting equation can be written each for positive and negative electric charge. However, only two of these three charge equations are linearly independent. As will be shown in Chapter 5, the positive and negative electric charge equations can be added together to generate the net charge equation.

Linear and angular momentum are three-dimensional vector quantities. Thus, three scalar, linearly independent equations can be written each for linear momentum and angular momentum.

Summary

In this chapter, we defined how engineering processes can be captured with mathematical statements in the forms of the accounting and conservation equations. To describe the system concerning the particular extensive property of interest, we characterized the system parameters and used these assumptions to find reductions to the governing mathematical equations, which can be in the algebraic, differential, or integral form. We also introduced the concepts of conservation; system, system boundary, and surroundings; open, closed, and isolated; reacting and nonreacting; energy interconversion; and steady-state and dynamic. These concepts and terms help define and reduce the Input, Output, Generation, Consumption, and Accumulation terms found in the accounting equation.

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Problems

For Problems 2.1–2.35, do the following:

- Draw a picture of the system.
- Name an extensive property that can be counted.
- Label the system, surroundings, and system boundary.
- State the time period of interest. Justify.
- Identify the system as open, closed, or isolated and state why.
- Identify the system as steady-state or dynamic and state why.
- Identify whether the system has a reaction and/or energy interconversion and state why.
- Should an algebraic, differential, or integral equation be used to describe the system? State why.
- Is the selected extensive property conserved in this system? State why.

(*Note*: There can be more than one correct answer; it depends on how you set up the system.) Problems are identified as pertaining to the areas of mass (M), energy (E), charge (C), or momentum (P).

2.1 (M) Blood is flowing through the heart. Consider the inlets to the heart as the pulmonary vein and the vena cava and the outlets from the heart as the pulmonary artery and the aorta. Ignore the coronary artery and cardiac veins. You want to write a model of the blood flowing through the heart that considers changes that occur in one second or less (i.e., a model that looks at different points in the cardiac cycle).

2.2 (P) Blood is flowing through the left side of the heart. The inlet is the pulmonary vein; the outlet is the aorta. Momentum is carried by the blood as it flows. Momentum is also added to the fluid through the force exerted by the heart as it pumps. You want to write a model that quantifies the momentum of the left side of the heart on the time scale of hours.

2.3 (M) A new drug is produced in a bioreactor. The process is termed a batch operation, since the reactants are added all at the beginning, the system is sealed and the reaction takes place, and then the products are removed. Within this processing system, the concentration of drug in the reactor increases as the reaction proceeds. Consider an accounting equation that tracks the drug only during the time that the reaction is occurring.

2.4 (M) The liver converts toxins to more innocuous constituents. Arteries and veins transport the blood containing the toxins into and out of the liver. The liver works continuously to detoxify materials, and the conditions of the liver do not change with time. Toxins do not accumulate in the liver. You are interested in writing an accounting equation on the toxins that are transformed in the liver.

2.5 (C) A cell in the kidney works constantly to keep the ion balance in the blood correct. The cell membrane contains ion pumps and channels that move Na^+ ions across the membrane. Looking specifically at one type of pump, Na^+ ions are transported from the inside of the cell to the outside of the cell. Assume that the cell does not generate or consume Na^+ ions. You are interested in writing a model on the positive charge contributed by Na^+ ions as they move across the cell membrane through the pumps.

2.6 (E) A calorimeter can determine a person's metabolic rate by measuring the quantity of heat liberated from the body in a given period of time. The calorimeter consists of a large air chamber with well-insulated walls that prevent energy transfer into or out of the calorimeter. As the person's body produces heat, a constant air temperature in the chamber is maintained by forcing the air through pipes in a cool water bath. A thermometer measures the temperature increase, which is used to calculate the rate of heat gain by the water bath, which equals the rate of heat release from the person's body. You want to measure your metabolic rate as you exercise in the calorimeter.

2.7 (E) A heart-lung bypass machine is used to circulate blood through the body during open heart surgery when the heart is stopped. Flowing blood enters and leaves the bypass machine. Special biocompatible material is used to line the walls of the machine so that no reactions occur in the blood. Energy in the forms of heat and mechanical work enters the bypass machine. During an operation, the bypass machine is maintained at constant temperature and other operating conditions. You are interested in writing an accounting equation on the total energy of the bypass machine.

2.8 (M) The effects of osmotic water shifts on red blood cells (RBC) can be observed experimentally by exposing the cells to hypertonic and hypotonic saline solutions. The inner fluid of RBCs is isotonic with 0.15 M NaCl. When RBCs are placed in a hypertonic solution, water leaves the cells, causing them to shrink. When RBCs are placed in a hypotonic solution, water enters the cells, causing rapid swelling, which may result in the bursting of some cells. An aliquot of 10^5 RBCs is added to 1 L of water with 0.05 M NaCl. Assume

that no metabolic activity is occurring in the RBCs to generate water. Yo are interested in writing an accounting equation on the water in the RBC a a function of time. Will the size of this system change?

2.9 (M) Poly(lactic acid), p(LA), is a biodegradable polymer currently approve by the FDA for use in humans as suture material. One limitation is tha p(LA) undergoes biochemical degradation in the body to lactic acid (LA which is acidic. If lactic acid is not cleared from the sutured area quickl enough, the local pH will be altered, potentially causing damage to the su rounding tissue. You design an in vitro experiment to determine the flui flow rate needed to remove enough lactic acid from the p(LA) site so as t alter the pH only one pH unit. Assume that p(LA) undergoes bulk erosion which means that the degradation rate is not constant. Assume that the loca flow rate of solution is constant. Answer the above questions for the prope ties of LA and p(LA).

2.10 (M) A hydrogel is a unique polymer network that absorbs water. To determin the equilibrium swelling content, the gel is first weighed dry. Then it is sub merged in water. Water is absorbed into the gel in a time-dependent manne After one hour, the gel is removed, the surface is blotted dry, and the sampl is reweighed. Absorption of water is not a chemical reaction. Answer th above questions for the properties of water and hydrogel during the proces of determining the equilibrium swelling content.

2.11 (M) A burn patient comes into a hospital emergency room. She is immediatel connected to an IV bag to replace the fluids she has lost during an accident an is currently losing by evaporation from her skin. Initially, you must administe IV fluid at a higher rate than she is currently losing it to replace the fluid lo in the accident. Once the patient is stabilized, you want to administer the flui at the same rate that she is losing it through evaporation. You are interested i writing an accounting equation to determine the rate of IV fluid replacement Answer the above questions for the initial time period and for the time afte she is stabilized.

2.12 (M) When a person gives blood, the blood flows from his body into a col lection bag for about half an hour. You are interested in writing a model t determine the blood in the person's body.

2.13 (E) Soon after you wake up on a mid-December morning, you grab a cup o hot coffee. You instantly feel the warmth of the mug in your hand. You woul like to know how the rate of energy flow from the cup to your hand change with time.

2.14 (M) Sprinters and long-distance runners rely on muscles to propel them a they run. Muscles require oxygen and glucose for contraction. Metabolism of these reactants generates carbon dioxide and lactate. Consider lactate i the adductor longus muscle in the leg of a sprinter during a 100-m race.

2.15 (C) Your roommate, a sociology major, does not understand the principles o heat conduction and grabs the metal handle of a pot of boiling water. The pain begins when outer nerve cells in the skin depolarize. This triggers an action potential along the axons of nerves until the signal reaches the brain. You are interested in modeling the signal conduction as a current from the hand to the brain during the time period before your roommate screams.

2.16 (M) Tumor cells are known for their rapid replication times. Using high resolution imaging equipment, you are able to estimate the number of cells i

the tumor. You are asked to determine the rate of growth of the tumor. Will the system boundary change size and/or shape?

2.17 (M) Nerve growth factor (NGF) is from the neurotrophin family of proteins that has been shown to have trophic effects on some cholinergic systems of the brain. Since it helps prevent neurons from dying, work has been done to see if NGF is viable as a therapeutic agent in the treatment of Alzheimer's disease or other neurodegenerative disorders. One possible treatment involves fabricating polymers into porous implants loaded with NGF powder and surgically placing them within the brain. Consider a model to track implanted NGF in the brain.

2.18 (M) Dialysis is a medical process that is used in patients experiencing lost kidney function. The primary role of a dialysis machine is to filter out waste and excess water from the blood. Dialysis works on the principle of diffusion across a semipermeable membrane. Blood flows out of the patient into a dialyzer and past a semipermeable membrane, on the other side of which dialysate fluid flows in the opposite direction. Waste substances will pass across the semipermeable membrane from an area of higher concentration in the blood into an area of lower concentration in the dialysate fluid. You are interested in writing a model that tracks the rate of removal of the waste compound urea from blood passing through the dialyzer. Assume that no urea passes from the dialysate back into the bloodstream.

2.19 (C) A blood pressure monitor is powered through a set of AA batteries. Each end of a battery has an electromotive force—that is, a force that it exerts on electrons. When the blood pressure monitor is on, the circuit of which the battery is part is closed, connecting the positive terminal (cathode) to the negative terminal (anode). As a result of the potential difference between the battery terminals across the circuit, a chemical reaction in the battery takes place and electrons are released from the anode. These electrons circulate from the anode to the cathode, thus generating a flow of charge, or current, through the circuit. You are interested in modeling the rate of electrons moving from one end to another of a battery that is discharging in this manner. Assume a time interval in which the battery maintains constant electromotive force

2.20 (P) Consider a red blood cell colliding with a platelet inside a capillary. You decide to model this collision in order to determine the transfer of momentum between the cells.

2.21 (P) A tennis player serves a ball over the net. During the serve, the tennis player accelerates her or his arm to give her or his racket substantial linear momentum. His or her racket then hits the ball as it falls. You decide to model the transfer of momentum from the racket to the tennis ball.

2.22 (P) Consider a continuous centrifuge of a plasmapheresis machine. Whole blood continuously travels from a patient undergoing plasmapheresis into the centrifuge. The centrifuge chamber rotates at a high velocity and transfers angular momentum to all the components of the blood. Components are separated by density into blood cells (red blood cells, white blood cells, and platelets) and plasma and continuously flow out of the centrifuge. You are interested in modeling the angular momentum of the red blood cells in the centrifuge.

2.23 (E) A defibrillator is used on a patient suffering from cardiac dysrhythmia, or irregular heart contractions. Once both pads are placed on the patient's chest,

electrical energy is transferred across the chest. This depolarizes the cardi muscle and encourages the heart's pacemaker to re-establish normal cardi muscle contractions. Model the flow of electrical energy through the patient chest.

2.24 (C) During the defibrillation process described in Problem 2.23, a certai amount of charge passes between the two pads and through the patient chest. Model the flow of current through the patient's chest.

2.25 (E) A small wind turbine is being used to recharge AA batteries. The wi turbine converts mechanical energy into electrical energy, which flows to th AA batteries to charge them. Create a model that tracks the amount of tot energy passing from the wind turbine into the batteries during an eight-hou period.

2.26 (C) After walking on a rug, you touch a metal door handle and feel a smal static discharge, or shock. The spark associated with static electricity i caused by the electrostatic discharge of excess charge flowing to its sur roundings. In this case, electrons were imparted to your body from th rug's fibers as you walked over it, giving you an excess negative charg This excess negative charge was then quickly transferred to the conductiv metal door handle, and you felt the shock of the resultant depolarization You are interested in modeling the transfer of charge in and/or out of you body during the shock.

2.27 (P) A pitcher winds up as he throws a fast ball. After releasing the ball, i travels at 88 mph until it reaches the catcher's mitt, where it is suddenly dece erated and comes to a stop. Create a model that tracks the linear momentum as the ball hits and subsequently stops in the catcher's mitt.

2.28 (M) Consider a plant undergoing photosynthesis. From equation [2.4-4], you can see that inputs of carbon dioxide and water are consumed in order t generate oxygen gas and glucose. You are interested in tracking the rate o carbon dioxide consumption in the photosynthetic process.

2.29 (E) A drop of saline falls from an IV bag into a collecting tube. Consider th conversion of potential energy into kinetic energy as this drop falls into th collecting tube. Create a model that describes the total energy of the salin drop immediately after it falls, but before it hits the rest of the liquid in th collecting tube. Hint: it may be convenient to consider the space between th IV bag and the bottom of the collecting tube as all or part of the system o interest.

2.30 (M) You leave mammalian cells in culture to incubate over a three-day period Upon returning, you find that the culture has been contaminated with yeast and a large number of mammalian cells have died as a result. You decide t create a model that describes the number of mammalian cells during the three day incubation period. Note that this mammalian cell line has a doubling time or the time necessary for the number of cells to double, of 1.5 days. (Pleas note that a Generation term can refer to self-generation or reproduction, and Consumption does not necessarily refer to actual consumption—it can mean death as well.)

2.31 (M) Consider the mammalian cell culture in the previous question. You decide to model the growth of yeast cells in the contaminated culture over the three day period.

2.32 (E) In electrical circuits, a capacitor is a two terminal component that stores electrical energy in the presence of a potential difference. Generally, a

electrical current flows through a capacitor, positive charge accumulates at one terminal while negative charge accumulates on the other, which creates a small electrostatic field that can store electrical energy. You are interested in creating a model that tracks how much electrical energy can be stored in a small capacitor. If you choose to include a source of electrical current in your system, assume that the electrical current flowing into the capacitor is from a battery, and remember that batteries operate on the principle of converting chemical energy to electrical energy.

2.33 (M) Consider a university (or any institute of higher learning) as your system. Choose any extensive property relating to campus activities that can be counted and tracked, such as the campus bookstore stock and sales tracking described in the chapter.

2.34 (M) Runners rely on muscles to propel them as they run. Muscles require a continuous supply of oxygen and glucose for contraction. Metabolism of these reactants generates carbon dioxide and lactate. Consider as the system the thigh muscle of a person during a 1000-m race. You wish to model glucose during the race.

2.35 (M) The small intestine is an important organ in the digestive system. With the help of various enzymatic secretions from the pancreas and liver, proteins, fats, and other nutritional components are broken down in the small intestine and are then absorbed through the walls of the small intestine into the blood. For example, proteins are enzymatically broken down into peptides. Consider as the system the small intestine of a person for the four hours following the start of dinner and track protein as the extensive property of interest.

2.36 List four extensive properties that are always conserved.

2.37 Why is the time scale important when determining whether a system is steady-state or dynamic? Consider your place of residence (your dorm, apartment, house, etc.). State a time period when your residence is dynamic. Justify. State a time period when your residence is steady-state. Justify.

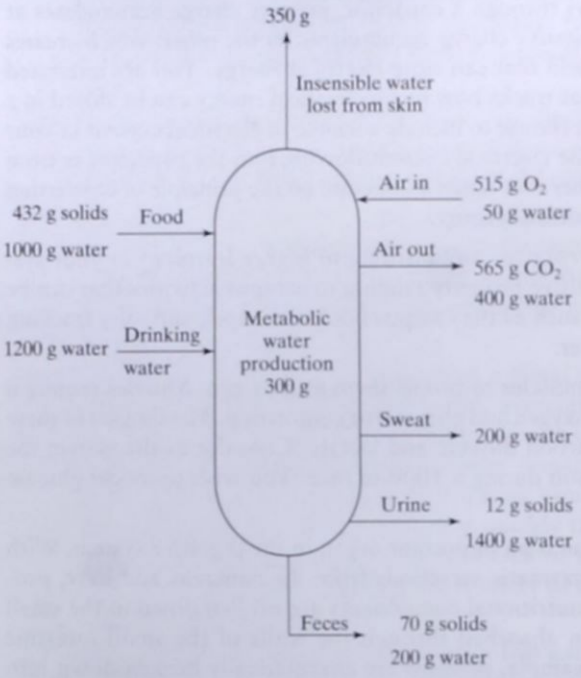
2.38 Water and solids enter and leave the human body through various means. The masses of water and solids for an average man for one day are shown in Figure 2.40.

(a) Write an algebraic accounting statement for the total solids in the system. Is the system open or closed, steady-state or dynamic, reacting or nonreacting? Calculate the total solids entering and leaving the system. Since Input does not equal Output, postulate an explanation.

(b) Write an algebraic accounting statement for water in the system. Is the system open or closed, steady-state or dynamic, or reacting or nonreacting? Plug the mass of water entering, leaving, and being generated in the system into the algebraic accounting statement. Can all the water be accounted for?

2.39 A patient in the hospital is being given a saline solution through an IV. Each day, she or he receives 1200 g of water through the IV. She or he receives water through no other means. A catheter collects all the urine leaving her or his bladder. It is determined that the daily water loss in the urine is 1600 g. Assume that water leaves her or his body through no other means. Metabolic activity is known to be normal.

While the patient is in the hospital for a week, the physician notices that she or he loses no weight. (From this, assume that the mass of water in the



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body does not change with time.) Yet, a quick mass balance on water that incorporates only the Inlet and Outlet terms doesn't make sense. Help the physician resolve this puzzle by defining whether the system is open or closed, steady-state or dynamic, or reacting or nonreacting. What is going on? Try to find some biological evidence to support your hypothesis. Finally, write the accounting equation with appropriate terms to describe the water balance in the patient.

2.40 For breakfast, Joe had 200 g oatmeal, 75 g milk, and 1 orange (225 g). For lunch, he had 1 apple (100 g), 4 pieces of bread (100 g), 90 g bacon, and 40 g cheese. For dinner, Joe had 350 g pork, 150 g asparagus, 150 g potatoes, and 2 pieces of bread (50 g). The protein, fat, and carbohydrate content as well as the energy content of the different foods are given in Table 2.6. You may use Excel, MATLAB, or another program of your choice for calculations.

(a) Assuming that the solids content is 30% and the water content is 70%, what is the total solids and water intake through food for Joe in one day? How does this value compare with that of the average man (Figure 2.4 in Problem 2.38)?

(b) Calculate the total grams of protein, fat, and carbohydrates derived from each food.

(c) Calculate the energy derived from each food based on the fuel values in Table 2.6.

(d) The physiologically available energy in food is as follows: carbohydrates–4 kilocalories/gram, fat–9 kilocalories/gram, and protein–4 kilocalories/gram. Using the total amount of protein, fat, and carbohydrates in Joe's diet, calculate the available energy from protein, fat, and carbohydrates. Average Americans receive 15% of their energy from protein, 40% from fat, and 45% from carbohydrates. Compare Joe to this average.

(e) How similar are the total energy values calculated in parts c and d?

TABLE 2.6

Protein, Fat, Carbohydrate, and Energy Content of Food*

Food	% Protein	% Fat	% Carbohydrate	Fuel value /100 g (kcal)
Apples	0.3	0.4	14.9	64
Asparagus	2.2	0.2	3.9	26
Bacon, broiled	25.0	55.0	1.0	599
Bread, white	9.0	3.6	49.8	268
Cheese	23.9	32.3	1.7	393
Milk, whole	3.5	3.9	4.9	69
Oatmeal	14.2	7.4	68.2	396
Orange	0.9	0.2	11.2	50
Pork, ham	15.2	31.0	1.0	340
Potatoes	2.0	0.1	19.1	85

*Data from Guyton AC and Hall JE, *Textbook of Medical Physiology*. Philadelphia: Saunders, 2000.

2.41 Consider Problem 2.17. It has been experimentally determined that the implant can release 0.24 μg NGF/day. In the brain, 0.11 μg NGF/day is eliminated due to metabolic processes and 0.023 μg NGF/day is lost due to nonspecific binding.

(a) Write a generic accounting equation to track NGF. Which (if any) terms can be eliminated?

(b) Neurons do not exhibit any reaction to NGF until the concentration reaches 2.0 ng/mL. What is the rate of accumulation of NGF in the brain? How long does it take before the entire brain (volume $\cong$ 1400 cm^3) reaches a therapeutic level? Assume perfect mixing and that the volume taken up by the implant is negligible.

(c) More realistically, you would want to treat only the diseased part of the brain. Alzheimer's disease is often associated with the death of cholinergic neurons in the basal forebrain (volume $\cong$ 400 cm^3). If the implant were placed in that region, how long would it take to reach 5.0 ng/mL?